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FROM

E. P. Seaver,
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7 April, 1893.

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A KEY

CONTAINING THE

ANSWERS TO THE EXERCISES

IN THE

FRANKLIN ELEMENTARY ALGEBRA

BY

EDWIN P. SEAVER, A.M.

SUPERINTENDENT OF PUBLIC SCHOOLS, BOSTON

AND

GEORGE A. WALTON, A.M.

AUTHOR OF WALTON'S ARITHMETICS, ARITHMETICAL TABLES, ETC.

PHILADELPHIA
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BOSTON.

ANSWERS.

SECTION I.

Article 9. PAGE 2.

1. $a + b$.
2. $a + b + c$.
3. $x + 80$.
4. $x + x$.

Art. 10. PAGE 3.

5. $8x$.
6. $11x$.
7. $7x$.
8. $5x$.
9. $a - b$.
10. $x - 3$.
11. $25 - x$.
12. $4x$.
13. $7x$.
14. $x - 7, 2x - 7$.
15. $4x$.
16. $7x$.

Art. 11. PAGE 4.

17. $8x$.
18. y.

Art. 13. PAGE 4.

19. $6x + 17$.
20. $2x + 3$.
21. $2x + 2$.
22. $9x$.
23. $6x - 8$.
24. $2x + 3$.
25. $3y - 1$.
26. $4x - 4$.

SECTION II.

Art. 17. PAGE 5.

1. 2.
2. 4.
3. 4.
4. 5.
5. 7.
6. 6.
7. 7.
8. 2.
9. 9.
10. 3.
11. 7.

Art. 19. PAGE 6.

12. Bat, 12 cents; ball, 48 cents.
13. Book, 50 cents; slate, 25 cts.
14. Worsted, 45 cents.
Muslin, 9 cents.
15. Before breakfast, 3 miles.
In the forenoon, 15 miles.
In the afternoon, 12 miles.

Art. 20. PAGE 7.

16. 9 of each.
17. 10 stamps.
18. 3 peaches.
6 pears.
19. 4 yards of red.
8 yards of blue.
20. Rode 2 hours.
Walked 8 hours.
21. William, 10 cts.
John, 20 cents.
Edgar, 60 cents.
22. 16 boys.
32 girls.
23. 9 to one boy.
27 to the other.
24. In the mud, 2 ft.
In water, 10 ft.
In the air, 12 ft.
25. 8 hours.
26. 5.

Art. 21. PAGE 8.

27. Horse, $270.
Harness, $45.
28. 351 and 27.
29. 1472 and 32.
30. Principal, $5000.
Interest, $200.
31. 432 and 24.
32. 12 pounds.
33. 40.
34. 9 days.
35. 22 hours.
36. House, $3600.
Barn, $1200.
Land, $9600.
37. Widow, $4000.
Each son, $2000.
Each daughter, $1000.
38. 8 oranges.
24 apples.
16 bananas.
39. 18 bush. barley.
36 bushels corn.
108 bushels oats.
40. 49.
41. 53.
42. A, 165 acres.
B, 82.5 acres.
C, 27.5 acres.
43. 17 men.
85 women.
34 children.
44. A, $600.
B, $1200.
C, $1800.
D, $3000.

Art. 26. PAGE 13.

45. 4.
46. 4.
47. 3.
48. 2.
49. 5.
50. 5.
51. 20.
52. 1.
53. 3.
54. $6\frac{1}{2}$.
55. 8.
56. 5.
57. 7.
58. 3.
59. 7.
60. 4.
61. 3.
62. 13.
63. 1.
64. 5.

Art. 28. PAGE 13.

65. 5 oranges.
66. 7 hours.
67. $x = 15$.
68. 16 years old.
69. $x = 25$.
70. 13 years old.

Art. 29. Page 14.

71. 12 coins.
72. John, 18 years.
Margaret, 9 yrs.
73. $2x-9$.
74. $3x-13$.
75. $3x-6$.
76. $4x-12$.
77. $5x-15$.
78. $4x-28$.
79. $12x-18$.
80. $3x+21$.
81. $10x-40$.
82. $30x+50$.

Art. 30. Page 15.

83. An orange, 5 cents.
A pear, 3 cents.
84. 2 and 9.
85. The first, 8 hours.
The second, 4 hours.
86. 12 one-dollar notes.
7 two-dollar notes.
87. Sugar, 10 cents.
Coffee, 35 cents.
88. Ralph, 13. Charles, 39.

Art. 31. Page 16.

89. A, $725.
B, $400.
90. 711 and 586 votes respectively.
91. 2146 and 1811.
92. 2346 and 2641.
93. 1522 miles.
94. 2832, 2239, and 718 votes respectively.
95. The parts are 5, $6\frac{1}{2}$, 9, and $12\frac{1}{2}$ feet long respectively.
96. In each hour, 43, 37, 31, 25, and 19 hills respectively.
97. The parts are 15, 17, 19, 21, 23, 25, and 27 inches long respectively.
98. 17, 14, 27, 8, and 33.
99. A piece of white, $41.25.
Of black, $51.25.
Of blue, $66.25.
100. 22, 7, and 12 gallons per minute respectively.
101. $1.25 and $0.50 a day respectively.
102. Better, 37 acres.
Poorer, 87 acres.
103. 38 ten-pound, and 22 sixteen-pound, balls.
104. $28\frac{4}{7}$ pounds at 35 cents.
$21\frac{3}{7}$ pounds at 42 cents.
105. 23 fives, 13 tens, and 51 twenties.
106. Eldest, $4900.
Next, $2500.
Next, $900.
Youngest, $300.

Art. 32. Page 18.

107. 10.
108. 12.
109. 10.
110. $3x$.
111. $7x$.
112. The numerator.
113. $9x$.
114. $12x$.
115. $3x+4x$.
116. $4x+9x$.
117. $6x-4x$.
118. $6x+4x-3x$.
119. $4x+x=60$.
120. $10x-3x=70$.
121. $x+\frac{3x}{4}=21$.
122. $4x+3x=84$.
123. $4x+3x=84$.

Art. 33. Page 19.

124. 4.
125. 4.
126. 14.
127. 6.
128. 18.
129. $53\frac{1}{3}$.
130. 2.
131. 90.
132. 6.
133. $1\frac{3}{8}$.

Art. 34. Page 20.

134. 25 boys, 30 girls.
135. $36.
136. 80 cents a yard.
137. 12 and 9 apples respectively.
138. The parts are 6 and 10 inches long respectively.
139. The parts are 4, 6, and 14 inches long respectively.
140. $1.20.
141. 6.
142. 18.
143. 12 years old and 7 years old respectively.
144. 40.
145. 20 yards.
146. 8.
147. 48 years.
148. 84 miles.

Art. 35. Page 21.

149. 2 hours.
150. $2\frac{2}{3}$ hours.
151. 6 days.
152. 4 days.
153. $2\frac{2}{9}$ hours.
154. 12 hours.
155. $1\frac{1}{3}$ days.

Art. 36. Page 22.

156. 17064.
157. 2471.
158. 4872.
159. $6\frac{3}{10}$.
160. 42.
161. $85.
162. 36 cents a yard.
163. 24 and 25.
164. 120 chestnuts.
165. 36 gallons.
166. 595 yards.
167. 1380.
168. 8700.
169. $884\frac{108}{113}$.
170. $3550.
171. $368.66 . . .
172. 5333.33\frac{1}{3}$.
173. $160.
174. $13500.
175. $6\frac{2}{3}$ %.
176. 120 chickens.
177. $4\frac{4}{5}$ hours.
178. $4\frac{2}{7}$ miles.
179. 2800 feet.
180. 14 miles.
181. $\frac{4}{5}$ of an hour.
182. $8\frac{2}{5}$ hours.
183. With the waste-pipe closed, $3\frac{1}{3}$ minutes; with it open, 20 minutes.
184. $2\frac{26}{47}$ days.

Art. 37. Page 25.

185. $x=3$.
186. $x=3$.
187. $x=2$.
188. $x=2$.

Art. 38. Page 25.

189. 4.
190. 5.
191. 2.
192. 3.
193. 3.
194. $2\frac{1}{4}$.
195. 7.
196. 3.
197. 2.
198. 3.
199. 6.
200. 4.
201. 5.
202. 5.
203. 7.
204. 3.
205. 5.
206. 18.
207. 4.
208. 6.
209. 12.
210. 15.

Art. 39. Page 26.

211. 5.
212. 3.
213. 10.
214. The father 42, the son 12 years old.
215. 28 and 20 years respectively.
216. $4 and $5 a yd. respectively.
217. 24 and 30 miles a day respectively.
218. Mother, 30 years old.
Daughter, 10 years old.
219. Father, 40 years old.
Son, 16 years old.
220. 45 shells. 8 in a row.
221. 4 companions. 30 pears.
222. In 8 years.
223. 6½ years ago.
224. 20 cents.
225. 27.
226. 20 gallons.
227. A, $18.
B, $27.
228. 48 trees.
229. 60.
230. 60 feet and 45 feet respectively.

Art. 40. Page 28.

231. 100 bushels.
232. 13 creditors. $3400.
233. 2½ cents apiece.
234. 14 dozen.
235. 8 owners. $40 a day.
236. 50 pounds. 28 cts. a pound.
237. $3600.
238. $450.
239. 60 gallons.
240. 126 gallons.
241. 264 acres.
242. $3000.
243. Whole property, $12000.
Each share, $3000.
244. 50.
245. 5 per cent.
246. 6 years and 24 years old respectively.
247. A, 42 years.
B, 48 years.
248. In 6 years.
249. In 15 days.
250. Half an hour.
251. 4½ pounds in one, 4 pounds in the other.
252. Lower part, 45 feet.
Whole mast, 70 feet.
253. 72 pounds.

Art. 41. Page 30.

254. $2x+3$.
255. $3+x$.
256. $4x+8$.
257. $2x+3$.
258. $3x+5$.

Art. 42. Page 31.

259. $\frac{x+1}{5}$.
260. $\frac{4x+9}{8}$.
261. $\frac{3x+21}{5}$.
262. $\frac{7x-7}{8}$.
263. $\frac{35-15x}{6}$.
264. $\frac{3-3x}{4}$.
265. 14.
266. 19.
267. 18.
268. 5.
269. 16.
270. 2.
271. 4.
272. 6.
273. 5.
274. 5.
275. 3.
276. 7.
277. 4.
278. 5.
279. 4.

Art. 43. PAGE 32.

280. 14.
281. $18 and $20 respectively.
282. 16 years and 20 years old respectively.
283. 5 years.
284. 3 gallons.
285. $20.
286. 60 years and 36 years respectively.
287. 4 feet.
288. 10 and 5 respectively.
289. 21 and 9.
290. 25 and 15 miles respectively.
291. 6.
292. 65.

Art. 44. PAGE 33.

293. 74.
294. 48 bushels.
295. 12, 13, and 14.
296. 17, 12, and 15 inches respectively.
297. 12 miles.
298. At a quarter past five o'clock.
299. 54 years.
300. 768 square rods.
301. 80 feet.
302. 44 inches.
303. 48 miles.
304. $3\frac{3}{4}$ hours down, $6\frac{1}{4}$ hours up; the stream running $1\frac{1}{15}$ miles an hour.

Art. 45. PAGE 35.

314. $4x+6$.
315. $5x+4$.
316. $4+x$.
317. $10x-3$.
318. $9x-12$.
319. $x-14$.
322. 5.
323. 8.
324. 3.
325. 3.
326. 1.
327. 3.
328. 4.
329. 7.
330. 10.
331. 1.

Art. 46. PAGE 36.

332. Robert, 7 cents. Henry, 6 cents.
333. 8.
334. The brother, 14. Albert, 10 years old.
335. 9 miles.
336. 4 beads.
337. 4 chickens.

Art. 47. PAGE 37.

338. 25.
339. 15 and 25.
340. In 6 years.
341. 6 cents.
342. A, 78 cents. B, 90 cents.
343. 14, 15, and 16.
344. $15000 at 4 %. $25000 at $4\frac{1}{2}$ %.
345. 20 and 60.
346. A, $120. B, $60.
347. 40 gallons and 20 gallons respectively.
348. $120.

Art. 48. PAGE 39.

349 Harness, $75.
Horse, $150.
Carriage, $450.

350 6 dimes.
8 nickels.

351 9 of each.

352 12 pounds.

353 $2.75.

354. 25 pounds at 9 cents, and 75 pounds at 13 cents.

355. 20 hundreds, 40 fifties, 80 twenties, 160 tens, 320 fives, 640 twos, and 1280 ones.

356. $20,000.

357. $19\frac{1}{21}$ and $\frac{20}{21}$.

358. 35 and 69.

359. 60 and 30.

360. 56 hours.
A, 84 miles.
B, 70 miles.

361. 2 miles.

362. At 11 o'clock A.M., day after to-morrow.

363. Cover, 20 ounces.
Other cup, 16 ounces.

364. A, in 26 days.
B, in $19\frac{1}{2}$ days.
C, in 13 days.

365. 20 dollars.

366. 5 and 6.

367. $8\frac{7}{16}$ miles down stream and back again.

368. 63 inches.

369. $x = 4$.

370. $x = 12$.

371. $x = 5$.

372. $x = 9$.

373. $x = 6.5$.

374. $x = 49500$.

375. $x = 10471.204\ldots$

376. $x = 4237.288\ldots$

377. $x = 408.371\ldots$

378. $x = 3000$.

379. $x = 200$.

380. $x = 9$.

381. $x = 72$.

382. $x = 19$.

SECTION III.

Art. 59. PAGE 46.

8. 4000 dollars.

9. -1000 dollars; that is, he *had lost* $1000 during the year.

10. 5000 dollars.

11. -5000 dollars; that is, he is 5000 dollars *in debt.*

12. 2500 dollars.

13. -200 dollars; that is, his "profit" was really a loss of $200.

14. He *saves* -160 dollars; that is, he has *run into debt* $160.

15. He *lost* -20 dollars; that is, he *gained* 20 dollars.

16. 3 paces.

17. -5 paces in *advance;* that is, 5 paces behind.

18. $+30°$, or 30° north latitude.

19. $-15°$, or 15° south latitude.

20. *Up* stream − 1 mile an hour; that is, in spite of their efforts they float *down* stream 1 mile an hour.
21. − 5 degrees, or 5 degrees below zero.
22. + 8 degrees, or 8 degrees above zero.
23. + 8, + 5, − 10.
24. − 10 miles *east*, and − 80 miles *north*; that is, 10 miles *west*, and 80 miles *south*.
25. 15 chains *north*, and 20 chains *west*.
26. He comes back to his starting point.
27. In − 2 years from the present time; that is, she *was* twice as old 2 years ago.
28. − 6 years hence; that is, 6 years ago.
29. − 24.
30. − 3 dollars *put in*; that is, 3 dollars must be *taken out* of each.
31. *Increased* by − 2 rods; that is, *diminished* by 2 rods.
32. A, − 1000 dollars.
B, + 2000 dollars.
A was in debt.
33. A, − 4000 dollars.
B, − 2000 dollars.
Both were in debt.

Art. 63. PAGE 50.

34. + 1.
35. − 2.
36. + 3.
37. − 8.
38. + 7.
39. − 3.
40. 0.
41. 0.
42. + 7000 dollars.
43. + 300 dollars.
44. − 200 dollars.
45. + 1000 dollars.
46. − 200 dollars.
47. − 500 dollars.
48. + 1.
49. − 1.
50. + 1.
51. + 15.
52. + 60.

Art. 66. PAGE 52.

53. + 11.
54. − 7.
55. + 8.
56. − 2.
57. + 2.
58. + 2.
59. − 3.
60. 0.
61. 0.
62. + 4000 dollars.
63. + 4000 dollars.
64. + 370.
65. *Risen* − 20°; that is, *fallen* 20°.
66. 65° northerly.
67. − 40° northerly, or 40° southerly.

Art. 69. PAGE 54.

68. − 35.
69. − 24.
70. + 54.
71. + 144.
72. + 1.
73. − 1.
74. − 1.
75. + 1.

Art. 72. PAGE 55.

76. − 4.
77. + 5.
78. − 9.
79. + 6.
80. + 1.
81. − 1.
82. − 1.
83. + 1.

Art. 82. Page 58.

84. $7abxy$.
85. $27ab^2c^2m$.
86. $-3abc$.
87. $-75m^2n^2$.
88. $-mn$.
89. $-mn$.
90. mn.
91. $-a^2$.
92. $4a$.
93. $9a^2b$.
94. $3a^4$.
95. $35m^5n^2$.
96. a^4.
97. m^4n^3.
98. $12mn$.
99. $a^3b^3c^3d^3e^3$.
100. $x^2y^2z^2w^2$.
101. $64a^3x^3$.
102. $16m^4y^4$.
103. $216x^3y^3$.
104. a^2.
105. $-m^3$.
106. a^4.
107. $-a^5$.
108. $+1$.
109. -1.
110. -1.
111. $+1$.
112. $+1$.
113. $648a^7b^7c^3$.
114. $-128a^3m^3x^4y^4$.
115. a^3.
116. $4a^2x^4y^6$.
117. $-4a^3m^6x^4y^2$.
118. $60k^9$.
119. n^{x+y+z}.
120. a^{m+2}.
121. a^{2m}.
122. $-40x^{p+q}$.
123. a^m.
124. a^n.
125. x^{p+1}.
126. $90a^2b^2cm^2$.
127. $-15abm^2n^2$.
128. 24.
129. $12m^6$.
130. $4a^5b^3$.
131. $-81ab^4mxy^3$.
132. $9h^6k^5l^2$.
133. $-a^6$.
134. a^6.
135. $1728a^6y^3$.
136. $256l^4m^4$.
137. x^{2p}.
138. $a^{3m}b^{3n}$.
139. $m^{rq}n^{sq}$.

Art. 85. Page 61.

140. am.
141. $-an$.
142. $-mn$.
143. an.
144. $2bn$.
145. $-1, +1$.
146. $a, -a, -a, a$.
147. abc.

148. x^5.
149. x^2.
150. $a^2 y^2 z^2$.
151. m^3.
152. $m^3 x$.
153. $2 x^2$.
154. $-5 a x^2$.
155. x^4.
156. $\frac{2b}{3}$.
157. $\frac{3}{y^2}$.
158. $\frac{a^2}{3}$.
159. $\frac{5}{z^4}$.
160. $\frac{3k^2}{5}$.
161. $-\frac{17klm}{3}$.
162. $\frac{4a^3x^4}{9}$.
163. $9 x^3 y$.
164. $4 m x$.
165. $-7 m n^3$.
166. x^{m-n}.
167. y^{m-1}.
168. m^q.
169. a^n.
170. p^{2n-3}.
171. x^{p-2}.
172. z^{1-n}.

Art. 89. Page 64.

173. $6 a x y$.
174. $3 l m$.
175. $k^2 l^3 m$.
176. $2 a^3$.
177. $4 y^2$.
178. $2 a x^2$.
179. $2 a x^2$.
180. $10 a$.
181. $9 m^2 y$.
182. $6 n z$.
183. $5 a m$.
184. $-18 a^2 x^5$.
185. $8 m^2 y$.
186. $-10 k l m$.
187. $x + y$.
188. $x - y$.
189. $x + y$.
190. $-l m - m n$.
191. $-l m + k l m$.
192. $a^3 + a^2$.
193. $3 y + y^3$.
194. $3 x^3 + 2 x^2 + 4 x$.
195. $7 a x^3 + 3 a^2 x^2 + 5 a^3 x$.
196. $4 a - 2$.
197. $a^4 - 4 a$.
198. $m^3 - 3$.
199. $a b - b$.
200. $-x y + 1$.
201. $3 a x y - 3 x$.
202. $4 a^2 x + 2 a x^2$.

Art. 93. Page 66.

203. $a^3 + 3a^2b + 3ab^2 + b^3$.
204. $a^3 + a^2x + ax^2 + x^3$.
205. $m^4 - m^3n + m^2n^2 - mn^3 + n^4$.
206. $k^4 + 2k^2l^2 + l^4$.
207. $x^4 - 2ax^3 + 4a^2x^2 - 8a^3x + 16a^4$.
208. $y^5 - 3y^4 + 6y^3 - 4y^2 + 3y - 1$.
209. $1 - x^3$.

Art. 96. Page 67.

210. $2x - 22$.
211. $-2x + y - 3z + 1$.
212. $-2x + 3y - 2z$.
213. $4x + 4z - 4$.
214. $2x$.
215. $x - y$.
216. $m + n$.
217. 0.
218. $-x - y - z$.
219. $2ax^2 + 4a^2x + 2x + 4a^3 - 3a$.
220. $x^5 + 6x^4 + 15x^3 + 19x^2 + 7x + 3$.
221. $x^3 + 4x^2 + 3$.
222. $4x^2 + 5xy + 4y^2 - 5x + 7y - 16$.
223. $3a^2 - 2ab - 2ac + 3b^2 - 2bc + 3c^2$.

Art. 99. Page 69.

224. $2y - 4$.
225. $5x^2 - 5x + 5$.
226. $-3x + 4y + 8$.
227. $-2a^2 - 2x^2$.
228. $x - 2$.
229. $y + b$.
230. $n - 2$.
231. -1.
232. $-l + 3m + n$.
233. $2b$.
234. $11a - 2b$.
235. $1 - x + y$.
236. $5a^2 - ab + xy$.
237. $2a^3 + 6ax^2$.
238. $x^4 + 4x^3y - 6x^2y^2 + 4xy^3 + y^4$.
239. $4x^4 + 7x^2 + 16$.
240. $\frac{1}{2}a - \frac{1}{2}b + \frac{1}{2}c$.
241. $\frac{1}{2}x - \frac{1}{2}y + \frac{1}{2}$.
242. $\frac{1}{3}ax - xy + 1$.

Art. 103. Page 71.

243. $2a^2 - ax$.
244. $2a + 4$.
245. $7a - 5b$.
246. a.
247. $4x$.
248. $2x^2 + 2$.
249. $x - y + z - w$.
250. $a + b + c$.
251. $8a - 3x$.
252. $a + b - c$.
253. $a - b + x - y$.
954. $3a - 3x$.
255. $l - m$.
256. $a + 4b$.
257. $-a + 2b$.
258. $13x - 9y$.
259. $5a - 3c$.
260. $2x - 2$.

Art. 105. Page 72.

261. $a - (-b + c - d)$.
262. $l - (m - n)$.
263. $w - (x + y - z)$.
264. $2k - (-l + m) + 2n$.
265. $a^2 - (b^2 - 2bc + c^2)$.
266. $x^5 - (4x^4 - 3x^3 + 2x^2) + x - 1$.

Art. 108. Page 75.

267. $3n - 6anx + 9bnx^2$.
268. $4ax^2yz + 2bxy^2z - 2cxyz^2$.
269. $-4abx + 6acx - 10bdx$.
270. $-3x^2y + 2x^2y^2 - 6xy$.
271. $3x^2 - 4xy + 8y^2$.
272. $a^2 - ax - 6x^2$.
273. $35x^2 - 33x + 4$.
274. $24a^2b^2 - 6a^2c^2$.
275. $2a^2b + 2a^2c - 3abc - ab^2 + b^2c$.
276. $-6ax^2 - 9bxy + 8axy + 12by^2$.
277. $l^2 - m^2 + 2mn - n^2$.
278. $10x^2 - 7xy - 19xz - 12y^2 - 29yz - 15z^2$.
279. $a^2x^2 + 2acxy + c^2y^2 - b^2 - 2bd - d^2$.
280. $1 + \frac{2}{3}b + \frac{1}{9}b^2 - \frac{1}{4}a^2$.
281. $a^6 - 3a^3x^2 + 2x^4$.
282. $8x^3 + 27$.
283. $18x^3 - 21x^2 - 21x + 10$.

284. $x^4 - 81$.
285. $x^4 + 4x^2 + 16$.
286. $y^7 + 3y^6 + y^5 - 3y^4 - 4y^3 - 3y^2 - 2y - 1$.
287. $a^4 - 1$.
288. $x^6 - 8x^5 + 20x^4 - 23x^3 + 24x^2 - 26x + 12$.
289. $a^4 + a^2x^2 + x^4$.
290. $a^6 - 3a^4b^2 + 3a^2b^4 - b^6$.
291. $y^4 - 2y^3 - 13y^2 + 14y + 24$.
292. $y^6 - m^6$.
293. $6x^{2m} - 7x^{m+n} - 3x^{2n}$.
294. $a^{2m} - x^{2m}$.
295. $a^{m+1} - a^m b - ab^m + b^{m+1}$.
296. $x^{n+2} + 7x^{n+1} + 18x^n + 16x^{n-1}$.

Art. 109. PAGE 77.

297. $a^2 - 2ab + b^2$.
298. $x^2 + 2x + 1$.
299. $a^3 + 3a^2b + 3ab^2 + b^3$.
300. $a^3 - 3a^2b + 3ab^2 - b^3$.
301. $y^4 - 4y^3 + 6y^2 - 4y + 1$.
302. $1 + 4a + 6a^2 + 4a^3 + a^4$.
303. $x^3 + 9x^2 + 27x + 27$.
304. $a^3 + 6a^2x + 12ax^2 + 8x^3$.
305. $a^3b^3 - 3a^2b^3c + 3ab^3c^2 - b^3c^3$.
306. $a^2 - 2ab + 2ac + b^2 - 2bc + c^2$.
307. $a^2 + 2ab - 2ac + b^2 - 2bc + c^2$.
308. $4m^2 - 12mn + 9n^2$.
309. $m^4 - 2m^2n^3 + n^6$.
310. $x^4 - 2x^3 + 3x^2 - 2x + 1$.
311. $2a^2 - 2xy$.
312. $4hx$.
313. $24x^2 + 128$.
314. $a^4 - 2a^2x^2 + x^4$.
315. $-4lm + 4ln$.
316. $x^4 - m^2x^2 + 2mx - 1$.
317. $2x^3 + 22x$.
318. $x^6 - 7x^4 + 32x^3 - 49x^2 + 48x - 9$.
319. $2a^3 - 2ax^2$.
320. $b^2 - d^2$.
321. $2l^2 + 2m^2 + 2n^2$.

322 $4x^2 + 4y^2 + 4z^2$.

323. $2n^3$.

324. $a^2b - ab^2 + b^2c - bc^2 + c^2a - ca^2$.

325. $3x^2 + 3y^2 + 3z^2 - 2xy - 2xz - 2yz$.

Art. 112. Page 81.

326 $a^2 - 5bx + 6x^2$.

327. $1 + 8ac - 2bc$.

328. $-2ax + 4b + 1$.

329. $-a + b - c + 2x$.

330. $c + d$.

331. $a + 2x$.

332. $3x - 5$.

333. $3 - b$.

334. $y^2 - y + 1$.

335. $a^4 + 3a^2 - 1$.

336. $5a^2 + 3x^2$.

337 $2ab$.

338. $a^3 + 2a^2b + 2ab^2 + b^3$.

339. $a^3 - a^2b - a^2x + abx + ax^2 - bx^2$.

340. $x - 4$.

341. $2a - 5x$.

342. $a + 1$.

343. $l - m + n$.

344. $5x + 4y + 3z$.

345. $a^2 - ab - ac + b^2 - bc + c^2$.

346. $3x^2 - x + 2$.

347. $2x^2 - 3xy + 4y^2$.

348. $ax + bx - y - z$.

349. $a - b - c$.

350. $x^2 - \frac{3}{4}x + 1$.

351. $\frac{1}{2}x^2 - \frac{3}{4}x + 6$.

352. $\frac{3}{2}x^3 - 5x^2 + \frac{1}{4}x + 9$.

353. $x^n + 2x^{n-1} - 3x^{n-2} + x^{n-3}$.

354. $x^m + 2x^n - 3x^p$.

355. $b^{n-1} - y^{n-1}$.

356. $a + x$.

357. $a^2 + ax + x^2$.

358. $a^3 + a^2x + ax^2 + x^3$.

359. There will be no remainder.

360. $a - x$.

361. $a^3 - a^2x + ax^2 - x^3$.
362. $a^5 - a^4x + a^3x^2 - a^2x^3 + ax^4 - x^5$.
363. There will be no remainder.
364. $a^2 - ax + x^2$.
365. $a^4 - a^3x + a^2x^2 - ax^3 + x^4$.
366. $a^6 - a^5x + a^4x^2 - a^3x^3 + a^2x^4 - ax^5 + x^6$.
367. There will be no remainder.
368. $16x^4 - 24x^3 + 36x^2 - 54x + 81$.
369. $a^2 - a + 1$.
370. $16k^2 - 12kl + 9l^2$.
371. $1 + 2x + 2x^2 + 2x^3 + \ldots$
372. $1 + 5x + 15x^2 + 45x^3 + \ldots$
373. $1 - x - x^2 + x^3 + x^4 - x^5 - x^6 + \ldots$
374. $1 - x + x^2 - x^3 + \ldots$
375. $1 + x + x^2 + x^3 + \ldots$
376. $1 + 2x + 3x^2 + 4x^3 + \ldots$
377. $1 - 2a + 2a^2 - 2a^3 + \ldots$
378. $a + ar + ar^2 + ar^3 + \ldots$
379. $a - ar + ar^2 - ar^3 + \ldots$
380. $1 - 2a + 3a^2 - 4a^3 + \ldots$
381. $1 + 4a + 8a^2 + 12a^3 + 16a^4 + \ldots$

SECTION IV.

Art. 119. Page 85.

1. $5a \times ab$, $5a^2 \times b$, $5 \times a^2b$.
2. $m \times m^2x^2$, $m^2 \times mx^2$, $m^3 \times x^2$, $mx \times m^2x$, $m^3x \times x$.
3. $4a^3y^2 \times 4a^3y^2$.
4. $5a^4$.
5. x^n.
6. $6a^2y \times 6a^2y \times 6a^2y$.
7. $12x^4y^3$.
8. $2ab^2$ is the sixth root.
9. $3m^2n \times 3m^2n \times 3m^2n \times 3m^2n$.
10. $5xy^2$.
11. $2m$.
12. A perfect square, because it can be separated into

two equal factors. A perfect cube, because it can be separated into three equal factors.

13. Yes: because 1 may be expressed as the product of any number of equal factors each equal to 1. Thus,

$$1 \times 1 \times 1 \times \ldots = 1.$$

14. $5my \times 3m \times 3m$.
15. $5ax \times 4x \times 4x$.
16. $16x^2y^6$.
17. $125a^3b^6$.
18. No. Yes; $a^2 \times (-1)$.
19. Yes.
$(-a)(-a)(-a) = -a^3$
$(-1)(-1)(-1) = -1$.
20. $-5kl^2$.
21. $-3m^4$.

Art. 123. Page 87.

22. $b(a+b)$.
23. $mn(m+n)$.
24. $3x(x-3)$.
25. $x(x^2+1)$.
26. $y^3(y^2-1)$.
27. $k^3(3k^2+5)$.
28. $5a^3x^3(x^2+a^2)$.
29. $5x(x^2+6x-2)$.
30. $5(5z^2-2z+1)$.
31. $abc(a-b+c)$.
32. $4ax(2x^2-3ax+a^2)$.
33. $x^2y^2(x^2-xy+y^2)$.
34. $3ab^2(a^2-2ab+b^2)$.
35. $3x^3y^2(x^2y-7x+9y^2)$.

Art. 126. Page 88.

36. $(a+b)(x+y)$.
37. $(a-b)(x+y)$.
38. $(x+a)(x+b)$.
39. $(x-y)(y-z)$.
40. $(x-3)(y+2)$.
41. $(a^2+1)(a+1)$.
42. $(2x^2-3)(3x+2)$.
43. $y^2(y^2-3)(y-2)$.
44. $(px+y)(qx-y)$.
45. $(m^2-n)(m+n^2)$.

Art. 129. Page 89.

46. $(x+a)(x+a)$.
47. $(x-a)(x-a)$.
48. $(x+3)(x+3)$.
49. $(x-6)(x-6)$.
50. $(x-13)(x-13)$.
51. $(y+1)(y+1)$.
52. $(z-20)(z-20)$.
53. $(2x+15)(2x+15)$.
54. $(x+5y)(x+5y)$.
55. $(5m-6y)(5m-6y)$.
56. $(a^2+x^3)(a^2+x^3)$.
57. $(x^3+12)(x^3+12)$.
58. $(x^5-32)(x^5-32)$.
59. $(a+x+m)(a+x+m)$.
60. $(x^3+y^5)(x^3+y^5)$.
61. $(x^m+y^n)(x^m+y^n)$.

Art. 132. Page 90.

62. $(x+5)(x+6)$.
63. $(x-3)(x-8)$.
64. $(x+3)(x-12)$.
65. $(x-7)(x+8)$.
66. $(x^2-8)(x^2+11)$.
67. $(z^3+1)(z^3+3)$.
68. $(y^3-1)(y^3+9)$.
69. $(6-x)(8-x)$, or
$(x-6)(x-8)$.
70. $(7-y)(8-y)$, or
$(y-7)(y-8)$.
71. $(xy-1)(xy-26)$.
72. $(yz+4)(yz+8)$.
73. $(a^2y^3-4)(a^2y^3-9)$.
74. $(m+n)(m+8n)$.
75. $(x+a)(x+c)$.
76. $(x-a)(x+c)$.
77. $(a+x+1)(a+x+2)$.

Art. 134. Page 91.

78. $(a+x)(a-x)$.
79. $(a+1)(a-1)$.
80. $(1+a)(1-a)$.
81. $(2x+3y)(2x-3y)$.
82. $(3y+4)(3y-4)$.
83. $(a^2+b^2)(a^2-b^2)$.
84. $(a^3+b^3)(a^3-b^3)$.
85. $(x^2+1)(x^2-1)$.
86. $(1+y^2)(1-y^2)$.
87. $(m^4+n^4)(m^4-n^4)$.
88. $(2k^2+4lm)(2k^2-4lm)$.
89. $(1+a^4x^4)(1-a^4x^4)$.
90. $(a+b-c)(a-b+c)$.
91. $(a+b+x+y)(a-b+x-y)$.
92. $(x+y+z)(x+y-z)$.
93. $(x-y+1)(x-y-1)$.
94. $(a+b-c)(a-b+c)$.
95. $(l+m-n)(l-m+n)$.
96. $(x^2+x-1)(x^2-x+1)$.
97. $(a+b+1)(a+b-1)$.

Art. 140. Page 93.

98. $a^6+a^5b+a^4b^2+a^3b^3+a^2b^4+ab^5+b^6$.
99. $a^7-a^6x+a^5x^2-a^4x^3+a^3x^4-a^2x^5+ax^6-x^7$.
100. $x^8-x^7y+x^6y^2-x^5y^3+x^4y^4-x^3y^5+x^2y^6-xy^7+y^8$.
101. $a^4+a^3+a^2+a+1$.
102. $1-x+x^2-x^3+x^4-x^5+x^6$.
103. $m^5-m^4+m^3-m^2+m-1$.

104. $1 + k^2 + k^4 + k^6.$
105. $1 - l\,m + l^2 m^2 - l^3 m^3 + l^4 m^4 - l^5 m^5.$
106. $m^4 - m^2 n^2 + n^4.$
107. $m^8 - m^6 n^2 + m^4 n^4 - m^2 n^6 + n^8.$
108. $x^6 + x^3 y^3 + y^6.$
109. $x^6 - x^3 y^3 + y^6.$
110. $(a^2 + x^2)(a + x)(a - x).$
111. $(y^2 + 1)(y + 1)(y - 1).$
112. $(9 + x^2)(3 + x)(3 - x).$
113. $(2 + x)(4 - 2x + x^2)(2 - x)(4 + 2x + x^2).$
114. $(c^4 + z^4)(c^2 + z^2)(c + z)(c - z).$
115. $(m^4 + 1)(m^2 + 1)(m + 1)(m - 1).$
116. $(a^2 + x^2)(a^4 - a^2 x^2 + x^4).$
117. $(a^4 + y^4)(a^8 - a^4 y^4 + y^8).$
118. $(m^6 - m^3 n^3 + n^6)(m^2 - m\,n + n^2)(m + n).$
119. $(y^6 + y^3 b^3 + b^6)(y^2 + y\,b + b^2)(y - b).$

Art. 147. Page 97.

120. $a\,p\,x.$
121. $3\,a^2 x.$
122. $7\,a^2 b.$
123. $d.$
124. $2\,a - b$
125. $a + b.$
126. $a + x.$
127. $a + b.$
128. $a^3 - a\,b.$
129. $m + n.$
130. $3\,x - 3.$
131. $x - 5.$
132. $x + 6.$
133. $x + 1.$
134. $x + 4.$
135. $x + c.$
136. $a^2 - x^2.$
137. $a - 3\,b.$

138. $x^2 + x.$
139. $a + b.$
140. $5.$
141. $31.$
142. $2\,x - 1.$
143. $4\,x - 5.$
144. $2\,a + 3\,x.$
145. $x^2 - 2.$
146. $x^2 + a\,x + a^2.$
147. $2\,x + 3.$
148. $x + 7.$
149. $a - b.$
150. $2\,x^3 - 4\,x^2 + x - 1.$
151. $x^2 + 5\,x + 1.$
152. $a - y.$
153. $12\,x - 5.$
154. $3\,a^2 + 2\,a\,b - b^2.$
155. $5\,x^2 - 1.$

Art. 152. PAGE 99.

156. abc.
157. $120a^4$.
158. $klmx$.
159. $bcx(a+x)$.
160. $(a^2+ax+x^2)(a+x)(a-x)$.
161. $ab^2(a+b)(a-b)$.
162. $(a+b)xy$.
163. $1-a^2$.
164. $16x^4-1$.
165. x^4-3x^2+2.
166. $x^3-6x^2-97x+630$.
167. m^3-3m^2-m+3.
168. $x^4+6x^3+3x^2-26x-24$.
169. $8(1-x)^2(1+x)(1+x^2)$.
170. $(a+b)^2(a-b)^2(a^2+ab+b^2)(a^2-ab+b^2)$.
171. $(a-b)(b-c)(c-a)$.
172. $(x-2)(x-3)(x-4)$.

SECTION V.

Art. 163. PAGE 103.

1. $\frac{4}{5}$.
2. $\frac{2}{3}$.
3. 126.
4. 560.
5. $\dfrac{a}{4bc^2}$.
6. $\dfrac{2lx}{ky}$.
7. $\dfrac{ay}{3x}$.
8. $1:5m$.
9. $b:1$.
10. $a:h$.
11. $3x:4y$.
12. $1+x:1$.
13. $\dfrac{2m+3}{b}$.
14. $\dfrac{m-n}{mn}$.
15. $\dfrac{m+n}{3}$.
16. $\dfrac{a}{x-a}$.
17. $\dfrac{6a}{5c}$.
18. $\dfrac{12m^3x^3+2m^2x^4}{18mn^2+3n^2x}$.
19. a^2+2x.

20. $\dfrac{a^2+x^2}{a-x}.$

21. $\dfrac{y+k}{y+m}.$

22. $\dfrac{x-1}{x+2}.$

23. $\dfrac{2m-1}{m+2}.$

24. $\dfrac{y^2-y+1}{y^2+y+1}.$

25. $\frac{10}{18}.$

26. $\dfrac{3amx^3}{2ax^2y}.$

27. $\frac{35}{40}.$ $\frac{1}{8}.$ $\frac{1}{40}.$

28. $\frac{15}{20}.$

29. $\dfrac{9m}{24}.$

30. $\dfrac{amx}{my}.$

31. 6. 18.

32. $b.$ $4b.$

33. $35x.$

34. $hm.$

35. $\dfrac{4k}{k}.$

36. $\dfrac{4x}{4}.$

37. $\dfrac{am}{bm}.$

38. $\dfrac{lyz}{xyz}, \dfrac{mxz}{xyz}, \dfrac{nxy}{xyz}.$

39. $\dfrac{abx}{a^2b}, \dfrac{by}{a^2b}, \dfrac{az}{a^2b}.$

Art. 167. Page 105.

40. $\frac{300}{420}, \frac{252}{420}, \frac{245}{420}.$

41. $\frac{3}{21}, \frac{21}{21}, \frac{49}{21}.$

42. $\dfrac{ayz}{xyz}, \dfrac{bzx}{xyz}, \dfrac{cxy}{xyz}.$

43. $\dfrac{2xy}{x^2y+xy^2}, \dfrac{x^2-y^2}{x^2y+xy^2}.$

44. $\dfrac{1-z}{1-z^2}, \dfrac{1+z}{1-z^2}, \dfrac{1}{1-z^2}.$

45. $\dfrac{ac-bc}{abc}, \dfrac{ab-bc}{abc}, \dfrac{ab-ac}{abc}.$

46. $\dfrac{2x-6}{6}, \dfrac{3x-6}{6}, \dfrac{6x-6}{6}.$

47. $\dfrac{a^2+2ab+b^2}{a^2-b^2}, \dfrac{a^2-b^2}{a^2-b^2}, \dfrac{a^2-2ab+b^2}{a^2-b^2}.$

48. $\dfrac{6}{6x}, \dfrac{3}{6x}, \dfrac{2}{6x}.$

49. $\dfrac{x^2}{x^3}, \dfrac{x}{x^3}, \dfrac{1}{x^3}.$

Art. 171. Page 107.

50. $1\frac{137}{140}$.

51. $1\frac{113}{168}$.

52. $3\frac{10}{21}$.

53. $\frac{kqr+lpr+mpq}{pqr}$.

54. $\frac{cp+bq+ar}{abc}$.

55. $\frac{11}{6a}$.

56. $\frac{a^2+a+1}{a^3}$.

57. 0.

58. $\frac{54x-13}{50}$.

59. $\frac{19x-23}{6}$.

60. The greater of the two quantities, a.

61. The smaller of the two quantities, b.

62. $\frac{2a^2}{a^2-b^2}$.

63. $\frac{a^2+b^2}{a^2-b^2}$.

64. $\frac{2x^2+2}{x^2-1}$.

65. $\frac{2x^2+7a^2}{x^2-ax-6a^2}$.

66. $\frac{1}{x+1}$.

67. $\frac{y}{x-y}$.

68. $\frac{x+5}{8}$.

69. $\frac{1}{2}$.

70. $\frac{4xy}{x^2-y^2}$.

71. $\frac{2x^2+x}{x^2+3x+2}$.

72. $\frac{x^2+y^2}{xy+y^2}$.

73. $\frac{h}{h-k}$.

74. $\frac{15}{16}$ exceeds $\frac{21}{23}$ by $\frac{9}{368}$.

75. $\frac{1}{19}$ of 20 exceeds $\frac{1}{20}$ of 21 by $\frac{1}{380}$.

76. $\frac{1}{6}$.

77. 0.

78. $\frac{x^3+x^2y+y^3}{(x+y)^3}$.

79. $\frac{24mn}{9m^2-4n^2}$.

80. $\frac{3a^2}{a^2-1}$.

81. $\frac{x+3}{x^4-1}$.

82. $\frac{y}{y+x}$.

Art. 174. PAGE 109.

83. $\frac{15}{16}$.
84. $\frac{3}{5}$.
85. $6:7$.
86. $5:2$.
87. $\frac{9}{2}$.
88. 18.
89. a.
90. $\frac{6ac}{b^2}$.
91. $\frac{4m}{z}$.
92. $\frac{2ab}{x}$.
93. $\frac{2uv}{5kl}$.
94. $\frac{ab^2}{c}$.
95. $\frac{6a^2mx^2}{7n}$.
96. $\frac{a^2c+a^2bc}{1+c}$.
97. $\frac{a^2}{a-z}$.
98. $\frac{k+l}{k-l}$.
99. $3m^2n-3mn$.
100. $\frac{y^2}{y^2-y+1}$.

Art. 177. PAGE 110.

101. $\frac{2}{9}$.
102. $\frac{2}{27}$.
103. $2:7$.
104. $2:15$.
105. $\frac{2}{21}$.
106. $\frac{3a}{4n}$.
107. $1+\frac{b}{c}$.
108. $3:80$.
109. $x:4y^2$.
110. $mx:k^3y$.
111. $\frac{a^2-ax+x^2}{a^2+x^2}$.
112. $\frac{a+b}{x^2-y^2}$.
113. $\frac{m}{2x^2}$.
114. $\frac{a+x}{a^2-2ax+x^2}$.
115. $\frac{m-2}{m^2+4m+4}$.
116. $\frac{x-2}{x^3+27}$.

Art. 180. PAGE 111.

117. $\frac{12}{5}$.
118. $\frac{15}{56}$.
119. $\frac{5}{12}$.
120. $4x^2$.

121. 1.

122. $\frac{1}{b\,m}$.

123. $\frac{a\,x^2}{6\,y^2}$.

124. 8 : 49.

125. 14 : 27.

126. $a\,k : b$.

127. $m\,p : n\,q$.

128. $\frac{k\,l}{k^2 - l^2}$.

129. $\frac{2\,a}{a - x}$.

130. $m\,k^2 + m$.

131. $\frac{a^3 - x^3}{a^3 + x^3}$.

132. $b^4 - b^2y^2 + y^4$.

133. $x^2 - 1$.

134. $2\,y^2 - 5\,y - 25$.

135. $\frac{a^2}{b^2}$.

136. $\frac{16\,x^4}{81\,y^4}$.

137. $\frac{x^2 + 2\,x\,y + y^2}{x^2 - 2\,x\,y + y^2}$.

138. $\frac{a^4 + 2a^2\,b^2 + b^4}{a^2\,b^2}$.

139. $\frac{a^8 + 4\,a^6 + 6\,a^4 + 4\,a^2 + 1}{a^4}$.

140. $\frac{x^4 + 2\,x^2\,h + h^2}{x^2}$.

141. $\frac{x^6 - 3\,x^4 + 3\,x^2 - 1}{x^3}$.

142. $\frac{m^2 + n^2}{m}$.

143. $\frac{3\,y}{4\,x}$.

144. $\frac{h^4 + h^2 + 1}{h^2}$.

145. $\frac{a^2\,b^2 + 2\,a\,b^2\,c + b^2\,c^2 - a^2\,c^2}{a^2\,b^2\,c^2}$, or $\frac{1}{a^2} - \frac{1}{b^2} + \frac{1}{c^2} + \frac{2}{a\,c}$.

146. $\frac{l^4 m\,n + l\,m^4 n + l\,m\,n^4 + l^3 m^3 + l^3 n^3 + m^3 n^3 + 3\,l^2 m^2 n^2}{l^2\,m^2\,n^2}$,

or $\frac{l^2}{m\,n} + \frac{m^2}{l\,n} + \frac{n^2}{l\,m} + \frac{l\,m}{n^2} + \frac{l\,n}{m^2} + \frac{m\,n}{l^2} + 3$.

Art. 185. PAGE 114.

147. 6.

148. 6.

149. $\frac{8\,a}{3}$.

150. $\frac{5}{6}$.

151. $\frac{1}{2}$.

152. $\frac{3}{7}$.

153. 15 : 2.

154. 5 : 8.

155. $\frac{5}{4}$.

156. 10 : 7.

157. $\frac{a^2}{b^2}$.

158. $\frac{3}{2}$.

Art. 194. PAGE 118.

191. Yes; for
$16 \times 114 = 38 \times 48$.
192. Yes; for $6 \times \frac{1}{6} = 2 \times \frac{1}{2}$.
193. $x = 231$.
194. $x = 275$.
195. $x = 150$.
196. $x = 242$.
197. $x = \frac{1}{6}$.
198. $1041\frac{13}{23}$ miles.
199. $a : b = x : y$
$a : x = b : y$
$y : b = x : a$, &c.

200. $x : y = 8 : 5$.
201. $x : y = 5 : 4$.
202. $x : y = 5$.
203. $x : y = 7 : 3$.
204. $x = 7$.
205. $x = 16$.
206. $x = 4$.
207. 3.
208. $x = 35$.
209. 42 miles.
210. The shares are $600 and $900.

211. 42 feet from one end.

212. The shares were $2100 and $2400.

213. $5\frac{3}{5}$ miles from one shore, $6\frac{2}{5}$ miles from the other.

214. 8 is a mean proportional between 4 and 16, also between 2 and 32.

215. 6 is not a mean proportional between 3 and 9, nor between 4 and 8; but it is a mean proportional between 4 and 9, likewise between 12 and 3.

216. $x = 10$.

217. $x = 6$.
218. $x = \frac{1}{6}$.
219. 9.
220. 12.
221. 15.

222. ab.
223. $a^2 - b^2$.
224. $a + b : c = c : a - b$.
225. 36 years.
226. 60 rods.

Art. 203. PAGE 123.

227. The first two terms of a proportion may be multiplied by any number, and the last two by any other number, and the result will be a proportion.

228. Both antecedents of a proportion may be multiplied by any number, and both consequents by any other number, and the result will be a proportion.

229. The same as **227**, changing "multiplied" to "divided."

230. The same as **228**, changing "multiplied" to "divided."

231. In any proportion, the sum of the first and second terms is to the second as the sum of the third and fourth is to the fourth.

232. In any proportion, the first term less the second is to the second as the third less the fourth is to the fourth.

233. In any proportion, the sum of the first and second terms is to the first term as the sum of the third and fourth is to the third.

234. In any proportion, the first term less the second is to the first as the third less the fourth is to the third.

235. In any proportion, the difference of the squares of the first two terms is to the square of the second term as the difference of the squares of the last two terms is to the square of the fourth term.

236. First prove (Art. 202) that $\frac{a^2}{A^2} = \frac{b^2}{B^2} = \frac{c^2}{C^2} = \ldots$ and to this result apply Art. 198.

237. Multiply the terms of each ratio by l, or m, or n, &c., and apply to the result Art. 198.

238. Apply the *test*, Art. 190.

Art. 208. PAGE 125.

239. 10 days.

240. $7\frac{1}{2}$ miles an hour.

241. $t : t' = 9 : 7$.

242. $160\frac{5}{6}$ feet. $321\frac{2}{3}$ feet.

243. 579 feet.

244. Its bulk would be doubled, provided the temperature were unchanged.

245. 45 inches.

246. $3\frac{1}{2}$ feet from one end, or $4\frac{1}{2}$ feet from the other.

247. $\frac{1}{200}$ of a pound.
248. From one $\frac{9}{34}$, and from the other $\frac{25}{34}$.
249. Equal to that of $16\frac{2}{3}$ wax candles.
250. $113\frac{7}{9}$ inches.

SECTION VI.

Art. 214. PAGE 130.

1. A's, $1835.00.
B's, $2752.50.
C's, $1651.50.

2. A's, $7314.00.
B's, $10971.00.
C's, $5851.20.
D's, $12433.80.

3. A's, $6164.38.
B's, $9246.58.
C's, $7089.04.
D's, $7500.00.

Art. 216. PAGE 131.

4. B's, $7200.
A's, $12500.
M's, $15600.

5. A, $138.12.
B, $176.80.
C, $185.08.

6. A, $9303.80.
B, $13291.14.
C, $7088.61.
D, $5316.45.

7. A's, $6550.63.
B's, $4367.09.
C's, $5240.51.
D's, $1091.77.

Art. 218. PAGE 133.

8. At $7\frac{1}{2}$ %.
9. At 5 %.
10. 3 years, 1 month, 15 days.
11. At simple interest, $35211.27.
12. At simple interest, $6106.87.
13. $109.83.
14. $657.965.
15. $(1.06)^5$ or 1.3382255776.
16. Problems 171, 172, 173, and 175 (pages 23 and 24).

Art. 221. Page 135.

17. $x = \frac{m}{a+b}$.

18. $x = \frac{b+m}{a}$.

19. $x = \frac{n-b}{a-m}$.

20. $x = \frac{cd-b}{ac-a}$.

21. $x = \frac{ab}{a+b}$.

22. $x = \frac{cm-nb}{an-cl}$.

23. $\frac{ma^2-mb^2}{2a}$.

24. $\frac{a-ak^2}{2}$.

25. $x = \frac{a}{2}$.

26. $x = \frac{(k+l)^2}{k-l}$.

27. $x = \frac{5a}{2}$.

28. The parts are $\frac{a-m}{2}$ and $\frac{a+m}{2}$. Problems 67, 68, 89, 90, 91, 92, 93 (pages 14 and 16).

29. The parts are $\frac{a}{1+m}$ and $\frac{am}{1+m}$. Problems 12, 13, and 14 (page 6); 27, 28, 29, 30 (pages 8 and 9); and 357 (page 39).

30. The parts are

$$\frac{a}{1+m+mn}, \frac{am}{1+m+mn}, \text{ and } \frac{amn}{1+m+mn}.$$

31. $\frac{al+bm+cn}{a+b+c}$.

32. $\frac{b-c}{n-m}$.

33. $\frac{amn}{m+n}$ miles.

34. He worked $\frac{c+bn}{a+b}$ days, and was idle $\frac{an-c}{a+b}$ days.

35. $\frac{a(r^2-r'^2)}{2r}$ miles.

36. The bank discount on a is art; the true discount is $\frac{art}{1+rt}$; and these differ by $\frac{ar^2t^2}{1+rt}$.

Art. 227. Page 140.

37. Let a have any finite value, and let r and r' have equal values. The infinite result indicates that the second body will never overtake the first, as evidently it cannot, going at the same rate as the first goes, and always at the distance a behind it.

38. Let $a = 0$, and r and r' have any equal values. The result $\frac{0}{0}$ indicates that the second body overtakes the first at the end of any time, however long or short, we choose to assign; for the two bodies are together at the start, and, going at the same rate, are always together.

39. $r = -5$, and $r' = 60$. The hands will be together at $55\frac{5}{13}$ minutes past 12 o'clock, again at $50\frac{10}{13}$ minutes past 1 o'clock; and so on once in every $55\frac{5}{13}$ minutes.

40. $\dfrac{cm - bm}{a - b}$ pounds at a cents. $\dfrac{am - cm}{a - b}$ pounds at b cents.

41. m pounds at a cents. 0 pounds at b cents. If one sort of tea is worth a cents a pound, and the mixture is to be worth a cents a pound, then *no* tea worth b cents a pound can be added, but m pounds of the other must be taken.

42. If $a = b = c$, both results would become indeterminate. The prices of both ingredients and of the mixture being the same, any quantity of either ingredient may be taken, provided only the quantity of both together shall not exceed m pounds.

SECTION VII.

Art. 239. Page 146.

1. $x = 1$
$y = 1.$

2. $x = 16$
$y = 7.$

3. $x = 7$
$y = 3.$

4. $x = 6$
$y = 10.$

5. $x = 4$
$y = 7.$

6. $x = 3$
$y = 10.$

7. $x = 7$
$y = -2.$

8. $x = -2$
$y = 19.$

9. $x = -5$
$y = 14.$

10. $x = 7$
$y = -5.$

11. $x = -2$
$y = 1.$

12. $x = -3$
$y = -2.$

13. $x = 36$
$y = 45.$

14. $x = 8$
$y = 9.$

15. $x = 144$
$y = 216.$

16. $x = 6$
$y = 2.$

17. $x = -22$
$y = 31\frac{1}{2}.$

18. $x = 8$
$y = 8.$

19. $x = \frac{s + d}{2}$
$y = \frac{s - d}{2}.$

20. $x = \frac{a s}{a + b}$
$y = \frac{b s}{a + b}.$

21. $x = \frac{a c - a b}{a c - b c}$
$y = \frac{a b - b c}{a c - b c}.$

22. $x = \frac{b' m - b m'}{a b' - a' b}$
$y = \frac{a m' - a' m}{a b' - a' b}.$

23.* $x = 2$
$y = 5.$

24.* $x = 4$
$y = 12.$

25.* $x = 3$
$y = 5.$

26.* $x = \frac{a b' - a' b}{b' m - b m'}$
$y = \frac{a b' - a' b}{a m' - a' m}.$

27. $x = 5$
$y = 9.$

28. $x = 12$
$y = 6.$

29. $x = 9$
$y = 4.$

30. $x = 2$
$y = 3.$

31. $x = 7$
$y = 4.$

32. $x = 7$
$y = 4.$

33. $x = 10$
$y = 11.$

34. $x = 10$
$y = 5.$

35. $x = 3$
$y = 5.$

36. $x = 6$
$y = 5.$

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* In this exercise it is best not to clear the equations of fractions.

Art. 240. Page 148.

37. Of a cow, \$25. Of an ox, \$35.

38. The poorer, 9 cents. The better, 11 cents.

39. John, 12 marbles. Henry, 18 marbles.

40. 8 gallons by one. 12 gallons by the other.

41. 50 hen's eggs. 30 duck's eggs.

42. The first, $2\frac{1}{5}$ cwt., or 220 pounds. The second, $2\frac{3}{5}$ cwt., or 260 pounds.

43. 14 and 6.

44. The father, 35 years. The son, 14 years.

45. 105 and 119.

46. $\frac{am}{m+n}$ and $\frac{an}{m+n}$.

47. Tea, 60 cents per pound. Sugar, 8 cents per pound.

48. Income, \$6000. Expense, \$4000.

49. Of the tea, 40 cents. Of the coffee, 15 cents.

50. $\frac{7}{13}$.

51. $\frac{7}{13}$.

52. $\frac{ak(a'-b')}{ab'-a'b}$ and $\frac{bk(a'-b')}{ab'-a'b}$. When k is added to each of these fractions, the results are $\frac{a'k(a-b)}{ab'-a'b}$ and $\frac{b'k(a-b)}{ab'-a'b}$.

53. By the larger, in 10 hours. By the smaller, in 15 hours.

54. 10 rods wide and 14 rods long.

55. 24 persons, each of whom paid \$1.75.

56. Principal, \$1250. Rate, 8 % per annum.

57. Principal, $\frac{bm-an}{m-n}$. Rate, $\frac{1200(a-b)}{bm-an}$.

Solve **56** by substituting in the results of **57**.

58 Arthur runs a mile in 10 minutes, and Philip in 11 minutes.

59. The first, 33 gallons. The second, 15 gallons.

60. The student, $3\frac{1}{3}$ miles an hour. The horse-cars, $6\frac{2}{3}$ miles an hour.

61. $14\frac{351}{356}$ pounds gold. $5\frac{5}{356}$ pounds silver.

62. $\frac{ss'(p+p')}{ps'+p's}$.

63. $\frac{100\,s(s'-s'')}{s''(s'-s)}$ and $\frac{100\,s'(s''-s)}{s''(s'-s)}$.

64. Of the wood, $\frac{wm'-w'm}{lm'-l'm}$. Of the metal, $\frac{wm'-w'm}{wl'-w'l}$.

65. 59 % pure milk. 41 % water.

66. Specific gravity of cast iron, 7.14 . . . Specific gravity of pine wood, 0.56 . . .

67. 33.7 %.

68. 10.315.

Art. 243. Page 154.

69. $x=4$
$y=1$
$z=5$.

70. $x=7$
$y=8$
$z=9$.

71. $x=8\frac{1}{2}$
$y=9$
$z=9\frac{1}{2}$.

72. $x=7$
$y=-10$
$z=9$.

80.* $w=0$
$x=5$
$y=7$
$z=10$.

73. $x=3$
$y=7$
$z=-4$.

74. $x=\frac{16}{25}$
$y=\frac{18}{25}$
$z=\frac{21}{25}$.

75. $x=62$
$y=46$
$z=34$.

76. $x=18$
$y=32$
$z=10$.

77. $x=\frac{a-b+c}{2}$
$y=\frac{a+b-c}{2}$
$z=\frac{-a+b+c}{2}$.

78. $x=12$
$y=60$
$z=60$.

79. $x=2$
$y=3$
$z=4$
$u=5$.

---

* A good method of elimination in such cases as this is to add all the given equations. From the result reduced to its simplest form subtract each given equation in turn. The several results are the values of the unknown quantities.

81.* $$w = \frac{3abcd}{-2bcd + acd + abd + abc}$$

$$x = \frac{3abcd}{bcd - 2acd + abd + abc}$$

$$y = \frac{3abcd}{bcd + acd - 2abd + abc}$$

$$z = \frac{3abcd}{bcd + acd + abd - 2abc}.$$

82. $x = -30$
$y = 31$
$z = -10.$

83. $x = -abc$
$y = ab + bc + ca$
$z = -(a + b + c).$

84. $v = 4$
$x = 3$
$y = 2$
$z = 1.$

85. $v = 3$
$w = 2$
$x = 2\frac{1}{2}$
$y = 4$
$z = 3\frac{1}{2}.$

Art. 244. Page 156.

86. Of a peach, 2 cts. Of a pear, 3 cts. Of an apple, 1 ct.

87. 24, 60, and 120.

88. A, \$3. B, \$4. C, \$8.

89. A, in 10 days. B, in 6 days. C, in 15 days. All, in 3 days.

90. A, in $\frac{2lmn}{lm + mn - nl}$ days. B, in $\frac{2lmn}{mn + nl - lm}$ days. C, in $\frac{2lmn}{nl + lm - mn}$ days.

91. 432.

92. Of the first, $32\frac{238}{891}$ pounds. Of the second, $14\frac{26}{891}$ pounds. Of the third, $3\frac{627}{891}$ pounds.

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* A good method of elimination in such cases as this is to add all the given equations. From the result reduced to its simplest form subtract each given equation in turn. The several results are the values of the unknown quantities.

93. 35 cents, 40 cents, 48 cents, and 56 cents, respectively.

94. \$1, \$5, \$10, and \$20, respectively.

95. A, in 108 days. B, in $49\frac{1}{11}$ days. C, in 27 days. D, in $18\frac{18}{29}$ days. All together, in $8\frac{4}{13}$ days.

96. The parts are 14, 28, 3, and 147.

97. The parts are $\frac{ab}{(1+b)^2}-b$, $\frac{ab}{(1+b)^2}+b$, $\frac{a}{(1+b)^2}$, and $\frac{ab^2}{(1+b)^2}$.

SECTION VIII.

Art. 254. Page 162.

1. $x^9+9x^8y+36x^7y^2+84x^6y^3+126x^5y^4+126x^4y^5+84x^3y^6+36x^2y^7+9xy^8+y^9$.

2. $m^7-7m^6n+21m^5n^2-35m^4n^3+35m^3n^4-21m^2n^5+7mn^6-n^7$.

3. $x^8-8x^7+28x^6-56x^5+70x^4-56x^3+28x^2-8x+1$.

4. $a^{10}-10a^9+45a^8-120a^7+210a^6-252a^5+210a^4-120a^3+45a^2-10a+1$.

5. $y^7-14y^6+84y^5-280y^4+560y^3-672y^2+448y-128$.

6. $243-405m+270m^2-90m^3+15m^4-m^5$.

7. $a^8-4a^6b^3+6a^4b^6-4a^2b^9+b^{12}$.

8. $16x^4-96x^3y+216x^2y^2-216xy^3+81y^4$.

9. $m^{15}-5m^{12}n^3+10m^9n^6-10m^6n^9+5m^3n^{12}-n^{15}$.

10. $a^{4m}+4a^{3m}b^m+6a^{2m}b^{2m}+4a^mb^{3m}+b^{4m}$.

11. $1-8k^2+28k^4-56k^6+70k^8-56k^{10}+28k^{12}-8k^{14}+k^{16}$.

12. $128+224x+168x^2+70x^3+\frac{35}{2}x^4+\frac{21}{8}x^5+\frac{7}{32}x^6+\frac{1}{128}x^7$.

13. $\frac{x^5}{243}+\frac{5x^4y}{162}+\frac{5x^3y^2}{54}+\frac{5x^2y^3}{36}+\frac{5xy^4}{48}+\frac{y^5}{32}$.

14. $\frac{x^6}{y^6} + \frac{6x^4}{y^4} + \frac{15x^2}{y^2} + 20 + \frac{15y^2}{x^2} + \frac{6y^4}{x^4} + \frac{y^6}{x^6}.$

15. $1 - \frac{7}{x} + \frac{21}{x^2} - \frac{35}{x^3} + \frac{35}{x^4} - \frac{21}{x^5} + \frac{7}{x^6} - \frac{1}{x^7}.$

16. $x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}.$

17. $a^6 + 12a^5bx + 60a^4b^2x^2 + 160a^3b^3x^3 + 240a^2b^4x^4 +$
$192ab^5x^5 + 64b^6x^6.$

18. $1 + 25z + 250z^2 + 1250z^3 + 3125z^4 + 3125z^5.$

19. $\frac{1}{m^8} - \frac{8}{m^7n} + \frac{28}{m^6n^2} - \frac{56}{m^5n^3} + \frac{70}{m^4n^4} - \frac{56}{m^3n^5} + \frac{28}{m^2n^6} -$
$\frac{8}{mn^7} + \frac{1}{n^8}.$

20. $\frac{32a^5l^{10}}{m^5} - \frac{20a^4l^8y}{m^2} + 5a^3l^6my^2 - \frac{5a^2l^4m^4y^3}{8} + \frac{5al^2m^7y^4}{128}$
$- \frac{m^{10}y^5}{1024}.$

Art. 256. Page 162.

21. $a^4 + 4a^3b + 4a^3c + 6a^2b^2 + 12a^2bc + 6a^2c^2 + 4ab^3$
$+ 12ab^2c + 12abc^2 + 4ac^3 + b^4 + 4b^3c + 6b^2c^2 + 4bc^3 + c^4.$

22. $a^3 + 3a^2bx + 3a^2cx^2 + 3ab^2x^2 + 6abcx^3 + b^3x^3 +$
$3ac^2x^4 + 3b^2cx^4 + 3bc^2x^5 + c^3x^6.$

23. $y^{10} + 10y^9 + 55y^8 + 200y^7 + 530y^6 + 1052y^5 + 1590y^4$
$+ 1800y^3 + 1485y^2 + 810y + 243.$

24. $a^3 - 3a^2b + 3a^2c - 3a^2d + 3ab^2 - 6abc + 3ac^2 +$
$6abd - 6acd + 3ad^2 - b^3 + 3b^2c - 3b^2d - 3bc^2 + 6bcd$
$- 3bd^2 + c^3 - 3c^2d + 3cd^2 - d^3.$

25. $8r^{12} + 36r^{11} + 102r^{10} + 207r^9 + 336r^8 + 441r^7 + 484r^6$
$+ 441r^5 + 336r^4 + 207r^3 + 102r^2 + 36r + 8.$

28. The fact upon which a rule may be based is thus stated: *The square of a polynomial is equal to the sum of the squares of the several terms plus double the products found by multiplying each term by the sum of the terms that precede* it.

29. $i^2 + 2ij + j^2 + 2(i+j)k + k^2 + 2(i+j+k)l + l^2 +$
$2(i+j+k+l)m + m^2 + 2(i+j+k+l+m)n + n^2.$

30. $p^2 + 2pq + q_2 + 2(p+q)r + r^2 + 2(p+q+r)s + s^2 + 2(p+q+r+s)t + t^2 + 2(p+q+r+s+t)u + u^2 + 2(p+q+r+s+t+u)v + v^2$.

Art. 266. Page 169.

31. 3657.
32. 7987.
33. 6660, with a remainder 6264.
34. 9229 with a remainder 9559.
35. 6310.
36. 14733, with a remainder 20512.
37. 8006.
38. 41621, with a remainder 15960.

Art. 270. Page 171.

39. $\frac{6}{8}+$.
40. $\frac{5}{10}+$.
41. $\frac{13}{20}+$.
42. $\frac{9}{10}+$.
43. $\frac{221}{1000}+$.
44. $\frac{601}{1000}+$.
45. 0.8.
46. 0.654.
47. 0.158.

Art. 273. Page 171.

48. 1.414 . . .
49. 1.732 . . .
50. 2.645 . . .
51. 3.872 . . .
52. 3.162 . . .
53. 5.099 . . .
54. 6.324 . . .
55. 9.486 . . .
56. 12.247 . . .

Art. 275. Page 172.

57. 1.264 . . .
58. 1.479 . . .
59. 2.154 . . .
60. 1.004 . . .
61. 2.412 . . .
62. 3.162 . . .
63. 1.968 . . .
64. 1.658 . . .
65. 2.299 . . .
66. 0.054 . . .
67. 0.1264 . . .
68. 0.9486 . . .
69. 0.3162 . . .
70. 0.0031 . . .
71. 0.1897 . . .

Art. 278. Page 174.

72. 1.4142136.
73. 1.7320508.
74. 2.2360680.
75. 2.8284271.
76. 3.3166248.
77. 3.7416574.
78. 4.1231056.
79. 5.4772256.
80. 7.0710678.
81. 8.6602540.
82. 8.9442719.
83. 12.6491106.

Art. 284. Page 176.

84. 5000.
85. 2000.
86. 900.
87. 400.

Art. 288. Page 179.

88. 423.
89. 5425.
90. 932.
91. 2504.
92. 211.
93. 709.
94. 8008.

Art. 290. Page 180.

95. Between $\frac{6}{8}$ and $\frac{7}{8}$.
96. Between $\frac{98}{100}$ and $\frac{99}{100}$.
97. Between $\frac{13}{16}$ and $\frac{14}{16}$.
98. Between 0.43 and 0.44.
99. Between 0.177 and 0.178.
100. 1.25992 . . .
101. 1.70997 . . .
102. 2.08008 . . .
103. 0.87358 . . .
104. 0.58480 . . .
105. 1.46478 . . .
106. 5.03968 . . .
107. 5.84803 . . .
108. 9.65489 . . .
109. 1.148 . . .
110. 0.492 . . .
111. 1.357 . . .
112. 1.379 . . .
113. 2.036 . . .
114. 0.973 . . .

Art. 296. Page 182.

115. $\pm 9a^2xy^3$.
116. $5my^2$.
117. $-7a^3x^4$.
118. $\pm\frac{3a^2x}{4by^3}$.
119. $-8a^2y^3$.
120. $\pm 10x^2y^3$.
121. $\frac{2m^2nx^2}{-5y^3}$.
122. $-9a^2x^4$.
123. $9a^4x^6$.
124. $\pm a^nb^{n-1}$.
125. a^pb^q.
126. $\pm\frac{(a+x)^nb^{n+1}}{ax^{n-1}}$.
127. $\pm 6x^{2n+1}y^m$.
128. $\pm 64b^{3n}y^6$.
129. $0.4m^2n^4$.
130. $\pm\frac{2}{5}x^2y^4z$.
131. $\pm\frac{1}{a^m}$.
132. $4z^4$.
133. $-64y^3z^6$.

Art. 298. PAGE 183.

134. $5\sqrt{3}$.
135. $2ab^2\sqrt{2ab}$.
136. $\frac{1}{2}\sqrt[3]{6}$.
137. $10\sqrt{-1}$.
138. $-4a^2b\sqrt[3]{2ab^2}$.
139. $6x^n\sqrt{2x}$.
140. $\frac{1}{3}x^2yz^2\sqrt{6xy}$.
141. $5\sqrt[4]{-2}$.
142. $ab^2\sqrt[5]{a^2b}$.
143. $\frac{1}{2}\sqrt{4n+2}$.

Art. 300. PAGE 183.

144. $\sqrt{45}$.
145. $\sqrt[3]{16}$.
146. $\sqrt{\frac{1}{3}}$.
147. $\sqrt{12x^{2n+1}}$.
148. $\sqrt{243a^7}$.
149. $\sqrt{\frac{2n+1}{2}}$.

Art. 305. PAGE 185.

150. $3a^2-2ax-2x^2$.
151. y^2-2y-3.
152. x^2-2x-3.
153. $3a^3+2a^2+a-1$.
154. x^3+2x^2-2x+4.
155. $3a^2-5ax+4x^2$.
156. $a+b+c$.

Art. 306. PAGE 186.

157. $$a^n+\frac{n}{1}a^{n-1}b+\frac{n(n-1)}{1\times2}a^{n-2}b^2+\frac{n(n-1)(n-2)}{1\times2\times3}a^{n-3}b^3+\frac{n(n-1)(n-2)(n-3)}{1\times2\times3\times4}a^{n-4}b^4+\frac{n(n-1)(n-2)(n-3)(n-4)}{1\times2\times3\times4\times5}a^{n-5}b^5+\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{1\times2\times3\times4\times5\times6}a^{n-6}b^6+\ldots$$

158. $+\frac{16\times15\times14\times13\times12\times11\times10\times9\times8}{1\times2\times3\times4\times5\times6\times7\times8\times9}x^7y^9$, or $+11440x^7y^9$.

159. $-\frac{15\times14\times13\times12\times11\times10\times9}{1\times2\times3\times4\times5\times6\times7}a^8b^7$, or $-6435a^8b^7$.

160. $+\frac{12\times11\times10\times9\times8\times7}{1\times2\times3\times4\times5\times6}x^6y^6$, or $+924x^6y^6$.

161. $\frac{m(m-1)\ldots(m-r+1)}{1\times2\times3\ldots(r-1)}a^{m-r+1}b^{r-1}$.

163. Yes. Because one of six equal factors has been found.

164. $6-2\sqrt{5}$.

165. $20+14\sqrt{2}$.

166. $97+56\sqrt{3}$.

167. $a\sqrt{a}+3a\sqrt{b}+3b\sqrt{a}+b\sqrt{b}$.

168. $1+3x+3x^2+x^3$.

169. 64.

170. 4. 32. 2.924. 27. 243. 11.18 . . . 128. 81. 3.659 . . . 3.343 . . .

SECTION IX.

Art. 310. Page 188.

1. $x=\pm 24$.
2. $x=39$.
3. $x=-67$.
4. $x=\pm 12$, or $\pm 12\sqrt{-1}$.
5. $x=\pm 5$.
6. $x=\pm 2$, or $\pm 2\sqrt{-1}$.
7. $x=\pm 2$.
8. $x=\pm\sqrt{a+b}$.
9. $x=\pm 2$.
10. $x=3$, $y=5$.
11. $x=4$, $y=7$, $z=12$.

Art. 312. Page 189.

12. $x=625$.
13. $x=1016.3$ . . .
14. $x=9$.
15. $x=5\sqrt{5}=11.18$ . . .
16. $x=1953125$.
17. $x=752.12$ . . .
18. $x=13.92$ . . .
19. $x=32$.

Art. 313. Page 189.

20. Length, 5.15 feet.
Width, 2.06 feet.
Depth, 2.355 feet.
21. 4550 men.
22. 63.
23. 72.
24. 491.
25. 20.
26. 50 and 15.
27. $\sqrt{pq}$ and $\sqrt{\frac{p}{q}}$.
28. 36 and 48.

29. $\sqrt{\frac{ap}{b}}$ and $\sqrt{\frac{bp}{a}}$.

30. 73 and 67.

31. $\sqrt{\frac{a+b}{2}}$ and $\sqrt{\frac{a-b}{2}}$.

32. 63 and 105.

33. $m\sqrt{\frac{a}{m^2+n^2}}$ and $n\sqrt{\frac{a}{m^2+n^2}}$.

34. $m\sqrt{\frac{a}{m^2-n^2}}$ and $n\sqrt{\frac{a}{m^2-n^2}}$.

35. 21, 35, and 56.

36. $a\sqrt[3]{\frac{p}{abc}}$, $b\sqrt[3]{\frac{p}{abc}}$, and $c\sqrt[3]{\frac{p}{abc}}$.

37. 6, 8, and 14.

38. 18.541 . . . feet.

39. 83.263 . . . feet.

40. Depth = diameter = 138.33 inches.

41. 8.486 . . . feet.

42. 12.407 . . . feet.

43. 483.60 square feet.

44. 21.00 . . . inches.

45. 5.64 . . . feet.

46. 11.75 . . . cubic feet.

47 4.05 . . . feet.

SECTION X.

Art. 321. Page 195.

1. $x_1 = 5$. $x_2 = 3$.

2. $x_1 = 4$. $x_2 = 3$.

3. $x_1 = 4$. $x_2 = -1$.

4. $x_1 = 8$. $x_2 = 2$.

5. $x_1 = 14.293 \ldots$
$x_2 = -13.293 \ldots$

6. $x_1 = 7$. $x_2 = -4$.

7. $x_1 = 8$. $x_2 = -5$.

8. $x_1 = 1.64 \ldots$
$x_2 = -3.64 \ldots$

9. $x_1 = 1.531 \ldots$
$x_2 = -6.531 \ldots$

10. $x_1 = 7.887 \ldots$
$x_2 = -0.887 \ldots$

11. $x_1 = 18.541 \ldots$
$x_2 = -48.541 \ldots$

12. $x_1 = 0.618 \ldots$
$x_2 = -1.618 \ldots$

13. $x_1 = 9$. $x_2 = -11\frac{2}{5}$.

14. $x_1 = 3$. $x_2 = -2\frac{1}{4}$.

15. $x_1 = 1.$ $x_2 = -0.7.$

16. $x_1 = 1.7037 \ldots$
$x_2 = -3.6537 \ldots$

17. $x_1 = 4.$ $x_2 = -2.$

18. $x_1 = 3.$ $x_2 = -\frac{1}{2}.$

19. $x_1 = 28.$ $x_2 = 9.$

20. $x_1 = 6\frac{9}{10}.$ $x_2 = 5.$

21. $x_1 = 21.$ $x_2 = 5.$

22. $x_1 = 3.$ $x_2 = 1\frac{81}{148}.$

23. $x_1 = 6.$ $x_2 = -4\frac{3}{4}.$

24. $x_1 = 2.$ $x_2 = -\frac{1}{3}.$

25. $x_1 = 9.$ $x_2 = 1\frac{2}{13}.$

26. $x_1 = 7.$ $x_2 = -1\frac{3}{7}.$

27. $x_1 = 10.$ $x_2 = -\frac{2}{3}.$

28. $x_1 = 14.$ $x_2 = -10.$

29. $x_1 = 6.$ $x_2 = 3\frac{1}{13}.$

30. $x_1 = 10.$ $x_2 = -1\frac{3}{4}.$

31. $x_1 = 5.$ $x_2 = -3.$

32. $x_1 = 7.$ $x_2 = -7\frac{2}{3}.$

Art. 322. Page 196.

33. Length, 60 feet; width, 40 feet.

34. 13 and 7, or -7 and -13.

35. $\frac{d}{2} \pm \frac{1}{2}\sqrt{4p + d^2}$ and $-\frac{d}{2} \pm \frac{1}{2}\sqrt{4p + d^2}$.

36. 29, or -30.

37 196, or 225.

38. 1024, or 961.

39. 121 yards long, 120 yards wide.

40. 12 and 13, or -13 and -12.

41. 3, 4, and 5, or -1, 0, and $+1$.

42. 15 and 16, or -16 and -15.

43. 6 and 12, or $-7\frac{1}{2}$ and -15.

44. \$70.

45. 48 rods long, 32 rods wide.

46. 480 feet long, 320 feet wide.

47. Hypothenuse, 17 feet; base, 15 feet; perpendicular, 8 feet.

48. 6.18 . . . and 3.819 . . . inches.

49. The parts are $-\frac{a}{2} \pm \frac{a}{2}\sqrt{5}$ and $\frac{3a}{2} \mp \frac{a}{2}\sqrt{5}$.

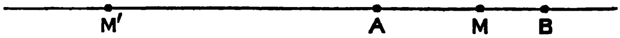
Note. The full interpretation of these results may be given in this way. Let an indefinite straight line be drawn, and two points,

A and B, marked in it at a distance apart equal to a. Then a point, M, can be found *between* A and B, so placed that

$$AM = -\frac{a}{2} + \frac{a}{2}\sqrt{5}, \text{ and } MB = \frac{3a}{2} - \frac{a}{2}\sqrt{5};$$

and another point, M', can be found outside of the part AB, so placed that

$$AM' = -\frac{a}{2} - \frac{a}{2}\sqrt{5}, \text{ and } M'B = \frac{3a}{2} + \frac{a}{2}\sqrt{5}.$$



50. 25 and 15.

51. $\frac{a}{2} \pm \sqrt{\frac{4s - a^3}{12a}}$ and $\frac{a}{2} \mp \sqrt{\frac{4s - a^3}{12a}}$.

52. 12 oranges.

53. 6 miles.

54. 8 horses.

55. A, 12 days; B, 6 days.

56. 96 barrels.

57. 28 men.

58. 75 sheep.

59. 30 cents a dozen.

60. 8 men, 12 women.

61. 12 miles, and 13 miles, an hour.

62. The first, 40 eggs; the second, 60 eggs.

63. The first 15, and the second 18, barrels; or the first 5, and the second 8, barrels.

64. A, 60 miles; B, 40 miles.

Art. 324. PAGE 201.

65. $x_1 = 7$. $y_1 = 5$.
$x_2 = 1\frac{2}{17}$. $y_2 = 8\frac{9}{17}$.

66. $x_1 = 1$. $y_1 = 2$.
$x_2 = \frac{1}{7}$. $y_2 = 2\frac{4}{7}$.

67. $x_1 = 3$. $y_1 = 2$.
$x_2 = -1\frac{2}{13}$. $y_2 = 4\frac{10}{13}$.

68. $x_1 = 5$. $y_1 = 3$.
$x_2 = 1\frac{7}{10}$. $y_2 = -\frac{3}{10}$.

69. $x_1 = 5$. $y_1 = 3$.
$x_2 = 2$. $y_2 = 6$.

70. $x_1 = 2\frac{1}{2}$. $y_1 = -3$.
$x_2 = -\frac{1}{2}$. $y_2 = 1$.

71. $x_1 = 3. \quad y_1 = 2.$
$x_2 = 3. \quad y_2 = -2.$
$x_3 = -3. \quad y_3 = 2.$
$x_4 = -3. \quad y_4 = -2.$

72. $x_1 = 7. \quad y_1 = 4.$
$x_2 = -7. \quad y_2 = -4.$

73. $x_1 = 3. \quad y_1 = 5.$
$x_2 = -3. \quad y_2 = -5.$

74. $x = 1\frac{1}{2}. \quad y = 1.$

75. $x_1 = 4. \quad y_1 = 2.$
$x_2 = 2. \quad y_1 = 4.$

76. $x_1 = 3. \quad y_1 = 2.$
$x_2 = -2. \quad y_2 = -3.$

77. $x_1 = 5. \quad y_1 = 4.$
$x_2 = -5. \quad y_2 = -4.$
$x_3 = 9\sqrt{\tfrac{1}{2}}. \quad y_3 = \sqrt{\tfrac{1}{2}}.$
$x_4 = -9\sqrt{\tfrac{1}{2}}. \quad y_4 = -\sqrt{\tfrac{1}{2}}.$

78. $x_1 = 7. \quad y_1 = 2.$
$x_2 = -7. \quad y_2 = -2.$
$x_3 = 5\sqrt{\tfrac{1}{2}}. \quad y_3 = -9\sqrt{\tfrac{1}{2}}.$
$x_4 = -5\sqrt{\tfrac{1}{2}}. \quad y_4 = 9\sqrt{\tfrac{1}{2}}.$

79. $x_1 = 5. \quad y_1 = 2.$
$x_2 = 2. \quad y_2 = 5.$
$x_3 = \tfrac{7}{2} + \tfrac{1}{2}\sqrt{89}.$
$y_3 = \tfrac{7}{2} - \tfrac{1}{2}\sqrt{89}.$
$x_4 = \tfrac{7}{2} - \tfrac{1}{2}\sqrt{89}.$
$y_4 = \tfrac{7}{2} + \tfrac{1}{2}\sqrt{89}.$

80. $x_1 = 4. \quad y_1 = 2.$
$x_2 = 2. \quad y_2 = 4.$
$x_3 = 3 + \sqrt{21}.$
$y_3 = 3 - \sqrt{21}.$
$x_4 = 3 - \sqrt{21}.$
$y_4 = 3 + \sqrt{21}.$

81. $x_1 = 6. \quad y_1 = 7.$
$x_2 = -6. \quad y_2 = -7.$
$x_3 = 7. \quad y_3 = 6.$
$x_4 = -7. \quad y_4 = -6.$

82. $x_1 = 10. \quad y_1 = 6.$
$x_2 = -10. \quad y_2 = -6.$

83. $x_1 = 11. \quad y_1 = 7.$
$x_2 = -11. \quad y_2 = -7.$

84. $x_1 = 30. \quad y_1 = 25.$
$x_2 = -87\tfrac{1}{2}. \quad y_2 = -8\tfrac{4}{7}.$

85. $x_1 = 4\tfrac{1}{2}. \quad y_1 = 6.$
$x_2 = -4\tfrac{1}{2}. \quad y_2 = -6.$
$x_3 = 18\sqrt{\tfrac{1}{34}}.$
$y_3 = -3\sqrt{\tfrac{1}{34}}.$
$x_4 = -18\sqrt{\tfrac{1}{34}}.$
$y_4 = 3\sqrt{\tfrac{1}{34}}.$

Art. 325. Page 202.

86. 6 and 20.
87. 1 and 18, or -18 and -1.
88. 17 and 15, or -17 and -15.
89. 14 and 8, or -14 and -8.
90. 12 and 4.
91. 12 feet long, 9 feet broad.

92. Base, 12 feet; perpendicular, 5 feet.

93. Base, 36 feet; hypothenuse, 39 feet.

94. 16 yards long and 2 yards wide, or 32 yards long and 1 yard wide.

95. Fore wheel, 11 feet; hind wheel, 15 feet.

96. 2, 5, and 9.

Art. 328. Page 203.

97. $z_1 = +2$. $z_2 = -2$. $z_3 = +1$. $z_4 = -1$.

98. $x_1 = +3$. $x_2 = -3$. $x_3 = +3\sqrt{-1}$. $x_4 = -3\sqrt{-1}$. $x_5 = +2$. $x_6 = -2$. $x_7 = +2\sqrt{-1}$. $x_8 = -2\sqrt{-1}$.

99. $x = 3$ or 1. See Ex. 127, further on.

100. $x_1 = +24$. $x_2 = -24$. $x_3 = +7$. $x_4 = -7$.

101. $y_1 = \sqrt[3]{12}$. $y_2 = \sqrt[3]{2}$. Other roots found by the process of Art. 330 are $y_3 = \sqrt[3]{12}\,(-\frac{1}{2} + \frac{1}{2}\sqrt{-3})$, $y_4 = \sqrt[3]{12}\,(-\frac{1}{2} - \frac{1}{2}\sqrt{-3})$, $y_5 = \sqrt[3]{2}\,(-\frac{1}{2} + \frac{1}{2}\sqrt{-3})$, and $y_6 = \sqrt[3]{2}\,(-\frac{1}{2} - \frac{1}{2}\sqrt{-3})$.

102. $z_1 = 3$. $z_2 = -2$. Other roots found in the solution of Ex. 126.

103. The roots are 1, -1, $\sqrt{-1}$, $-\sqrt{-1}$, $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{1}{2}\sqrt{-1}$, and $-\frac{1}{2}\sqrt{-1}$.

104. The roots are $\frac{5}{13}$, $-\frac{5}{13}$, $\sqrt{-1}$, and $-\sqrt{-1}$.

105. The roots are $\sqrt{3 + 3\sqrt{6}}$, $-\sqrt{3 + 3\sqrt{6}}$, $\sqrt{3 - 3\sqrt{6}}$, and $-\sqrt{3 - 3\sqrt{6}}$.

106. The roots are $\sqrt[n]{4}$ and $\sqrt[n]{-\frac{3}{2}}$.

107. The roots are $a + b$, $-(a + b)$, $a - b$, and $-(a - b)$.

Art. 330. Page 205.

108. $(x - 3)(x + 1) = 0$. $x_1 = 3$. $x_2 = -1$.

109. $(x + 5)(x + 6) = 0$. $x_1 = -5$. $x_2 = -6$.

110. $(x - 7)(x + 8) = 0$. $x_1 = 7$. $x_2 = -8$.

111. $(y + 56)(y - 1) = 0$. $y_1 = -56$. $y_2 = 1$.

112. $(y - 1)(y + 1)(y - \sqrt{2})(y + \sqrt{2}) = 0$.
$y_1 = 1$. $y_2 = -1$. $y_3 = \sqrt{2}$. $y_4 = -\sqrt{2}$.

113. $(y^2+11)(y^2-8)=0.$

$y_1=\sqrt{-11}.\quad y_2=-\sqrt{-11}.\quad y_3=\sqrt{8}.\quad y_4=-\sqrt{8}.$

114. $(y+a)(y+c)=0.\quad y_1=-a.\quad y_2=-c.$

115. $(y-a)(y+c)=0.\quad y_1=a.\quad y_2=-c.$

116. See Ex. 107, above.

117. $5,\ -\frac{5}{2}+\frac{5}{2}\sqrt{-3},\ -\frac{5}{2}-\frac{5}{2}\sqrt{-3}.$

118. $-5,\ \frac{5}{2}+\frac{5}{2}\sqrt{-3},\ \frac{5}{2}-\frac{5}{2}\sqrt{-3}.$

119. $6,\ -3+3\sqrt{-3},\ -3-3\sqrt{-3}.$

120. $-\frac{1}{2},\ \frac{1}{4}+\frac{1}{4}\sqrt{-3},\ \frac{1}{4}-\frac{1}{4}\sqrt{-3}.$

121. $a,\ -\frac{a}{2}+\frac{a}{2}\sqrt{-3},\ -\frac{a}{2}-\frac{a}{2}\sqrt{-3}.$

122. $-a,\ \frac{a}{2}+\frac{a}{2}\sqrt{-3},\ \frac{a}{2}-\frac{a}{2}\sqrt{-3}.$

123. $2,\ -1+\sqrt{-3},\ -1-\sqrt{-3},\ -1,\ \frac{1}{2}+\frac{1}{2}\sqrt{-3},\ \frac{1}{2}-\frac{1}{2}\sqrt{-3}.$

124. $3,\ -\frac{3}{2}+\frac{3}{2}\sqrt{-3},\ -\frac{3}{2}-\frac{3}{2}\sqrt{-3},\ -3,\ \frac{3}{2}+\frac{3}{2}\sqrt{-3},\ \frac{3}{2}-\frac{3}{2}\sqrt{-3}.$

125. $1,\ -\frac{1}{2}+\frac{1}{2}\sqrt{-3},\ -\frac{1}{2}-\frac{1}{2}\sqrt{-3},\ -1,\ \frac{1}{2}+\frac{1}{2}\sqrt{-3},\ \frac{1}{2}-\frac{1}{2}\sqrt{-3}.$

126. $3,\ -\frac{3}{2}+\frac{3}{2}\sqrt{-3},\ -\frac{3}{2}-\frac{3}{2}\sqrt{-3},\ -2,\ 1+\sqrt{-3},\ 1-\sqrt{-3}.$

127. $3,\ -\frac{3}{2}+\frac{3}{2}\sqrt{-3},\ -\frac{3}{2}-\frac{3}{2}\sqrt{-3},\ 1,\ -\frac{1}{2}+\frac{1}{2}\sqrt{-3},\ -\frac{1}{2}-\frac{1}{2}\sqrt{-3}.$

128. $x^2-8x+15=0.$
129. $x^2-3x-28=0.$
130. $20x^2-27x+9=0.$
131. $4x^2-36x+77=0.$
132. $x^2-25=0.$
133. $x^2-4x+1=0.$
134. $x^2-10x-20=0.$
135. $x^2-2ax+a^2-b=0.$
136. $x^2-2kx+k^2-l^2=0.$
137. $4x^2-4hx+h^2-m=0.$
138. $x^2+2x+4=0.$
139. $x^2+2x+2=0.$
140. $x^2-2sx+p^2=0.$

For further exercises, take the roots found in Arts. 328 and 330, and with them reconstruct the equations to which they belong.

SECTION XI.

Art. 341. Page 210.

1. 9.
2. $\frac{64}{3375}$.
3. 27.
4. $\frac{1}{256}$.
5. $41\frac{2}{3}$.
6. 729.
7. $\frac{4096}{729}$.
8. 64.
9. 64.
10. a.
11. a^5.

12. a^{-2}.
13. x^{-5}.
14. x^2.
15. $-12x$.
16. $12axy^{-2}$.
17. $16x^{p+q}$.
18. $a^{-3}x^{-2}$.
19. $a^{-3}x^2$.
20. $2y^{-k}$.
21. a^{-1}.
22. $\frac{2}{9}k^2$.

23. $15y^{-3}$.
24. 9.
25. -25.
26. apx^{p-a}.
27. k^l.
28. k^{3l}.
29. z^m.
30. $-a^2$.
31. $a^{-m}b^{x-y}$.
32. $20ax^{-2}y$.
33. $-2x^{-3}y^4z^{-2}$.

34. a^{2k}. The square of that quantity.

37. $\frac{x}{y^2}-\frac{2x^{\frac{1}{2}}}{yz^{\frac{1}{2}}}+\frac{1}{z}$.

39. $27a^6x^{-9}$.
40. $\frac{1}{27}x^{-6}y^3$.
41. 27.
42. 216.
43. $\frac{1}{64}$.
44. $\frac{11}{24}$.
45. $\frac{1}{8}$.
46. 25.
47. 405.

48. 28.
49. $\frac{1}{27}$.
50. $y^{\frac{5}{6}}$.
51. $a^{\frac{23}{20}}$.
52. $a^{\frac{7}{20}}$.
53. 20.
54. $\frac{1}{2}a^{\frac{1}{3}}b^{\frac{2}{3}}$.
55. $-18a^{\frac{1}{3}}b$.

56. $x^{\frac{n+m}{nm}}$.
57. $x^{\frac{n-m}{nm}}$.
58. $x^{\frac{rv+us}{sv}}$.
59. $x^{\frac{rv-us}{sv}}$.
60. $\frac{1}{64}a^4b^9$.
61. $\frac{1}{4}a$.

62. $\frac{1}{64}x^{-\frac{1}{2}}y$, or $\frac{y}{64\sqrt{x}}$.
63. $x^{\frac{ur}{vs}}$.
64. $\frac{1}{8x^2}$.
65. $a^{\frac{1}{2}}$.

66. $b^{\frac{2}{5}}y^{-\frac{6}{5}}$, or $\sqrt[5]{\frac{b^2}{y^6}}$.
67. x^2.
68. 7^4+4^3, or 2465.
69. -32712.
70. 2^3, or 8.
71. 123 × 456, or 56088.

Art. 343. PAGE 212.

72. $a^{-1} - x^{-1} - a x^2 + a^2 x$.

73. $a^2 b^{-2} + 1 + a^{-2} b^2$.

74. $a^2 b^{-2} + 2 + a^{-2} b^2$.

75. $x^3 y^{-3} - 3 x y^{-1} + 3 x^{-1} y - x^{-3} y^3$.

76. $x^3 - x - x^{-1} + x^{-3}$.

77. $x y^{-1} - 1 + x^{-1} y$.

78. $a^{-1} x^{-1} - a x$.

79. $x y^{-1} + 1 + x^{-1} y$.

80. $a^{\frac{3}{2}} - b$.

81. $a^{\frac{3}{2}} + x^{\frac{3}{2}}$.

82. $a^2 + a x + x^2$.

83. $a^3 - a^{\frac{4}{3}} + 2 a^{\frac{1}{3}} - 2 - a^{-\frac{2}{3}} + a^{-3}$.

84. $x^{\frac{1}{2}} - y^{\frac{1}{2}}$.

85. $a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} + b^{\frac{2}{3}}$.

86. $m^{\frac{3}{4}} - m^{\frac{1}{2}} n^{\frac{1}{4}} + m^{\frac{1}{4}} n^{\frac{1}{2}} - n^{\frac{3}{4}}$.

87. $k^{\frac{4}{3}} - k^{\frac{2}{3}} l^{\frac{2}{3}} + l^{\frac{4}{3}}$.

88. $a^{4.5} - a^3 b^{1.5} + a^{1.5} b^3 - b^{4.5}$.

89. $a^{\frac{3}{4}} + a^{\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{4}} b + b^{\frac{3}{2}}$.

90. $a^{0.4} - a^{2.5}$.

91. $a^{-\frac{1}{2}} y^{\frac{1}{2}} - 1 + a^{\frac{1}{2}} y^{-\frac{1}{2}}$.

Art. 352. PAGE 215.

92. $15\sqrt{7}$.

93. $4\sqrt{2} + 3\sqrt{3} + 2\sqrt{5}$.

94. $6\sqrt{3}$.

95. $\sqrt{a} + \sqrt{b}$.

96. $\sqrt{a} + \sqrt[3]{a}$.

97. $5\sqrt{a}$.

98. $\sqrt[6]{a^5}$.

99. $6 a$.

100. $\sqrt[6]{5^5}$.

101. $\sqrt{5}$.

102. $\frac{3}{2}\sqrt[6]{2}$.

103. $\sqrt[12]{y^{29}}$.

104. $\sqrt[12]{y^{11}}$.

105. $\sqrt[mn]{x^{np+mq}}$.

106. $\sqrt[mn]{x^{np-mq}}$.

107. $a y \sqrt{b^2 x^2 y}$.

108. $15 a^3 m^2 \sqrt[6]{a^5 m}$.

109. $20 m^2 y^3 \sqrt[6]{m y^5}$.

110. $20 k^2 l^2$.

111. $\sqrt[5]{y^2}$.

112. $a^2 - 3\sqrt[6]{a^{17}} + 3\sqrt[3]{a^{11}} - \sqrt{a^9}$.

113. $a^{\frac{3}{m}} - 3\,a^{\frac{2}{m}}\,b^{\frac{1}{n}} + 3\,a^{\frac{1}{m}}\,b^{\frac{2}{n}} - b^{\frac{3}{n}}$, or

$\sqrt[m]{a^3} - 3\sqrt[m]{a^2}\sqrt[n]{b} + 3\sqrt[m]{a}\sqrt[n]{b^2} - \sqrt[n]{b^3}$.

114. $a^{\frac{2}{n}} - 2\,a^{\frac{1}{n}}\,b^{\frac{1}{n}} + b^{\frac{2}{n}}$, or $\sqrt[n]{a^2} - 2\sqrt[n]{a\,b} + \sqrt[n]{b^2}$.

115. $a^{\frac{2m}{n}} + 2\,a^{\frac{m^2+n^2}{mn}} + a^{\frac{2n}{m}}$, or $\sqrt[n]{a^{2m}} + 2\sqrt[mn]{a^{m^2+n^2}} + \sqrt[m]{a^{2n}}$.

116. $\frac{1}{a}$.

117. $\sqrt{x}$.

118. $\frac{1}{\sqrt[6]{a\,b^2}}$.

119. $\sqrt{a}$.

120. $\sqrt{a}$.

121. $\sqrt[12]{5^4 \times 7}$, or $\sqrt[12]{4375}$.

122. $\sqrt[6]{135}$.

123. $\sqrt[3]{a}$, $\sqrt[n]{a}$.

124. 1.587 . . .

125. $1\frac{3}{5}$.

126. 0.00606 . . .

127. 0.128 . . .

128. $\sqrt[3]{2}$.

129. $\frac{3}{20}\sqrt[3]{100}$.

130. $\frac{3}{5}\sqrt[3]{25}$.

131. $\frac{1}{8}\sqrt[3]{4}$.

132. $\sqrt{2}$.

133. $\sqrt[3]{9}$.

134. $2(\sqrt{3} - \sqrt{2})$.

135. $\sqrt{5} - \sqrt{2}$.

136. $\sqrt{5} + 1$.

137. $\frac{2\sqrt{2} + \sqrt{3}}{5}$.

138. $\frac{3 + \sqrt{3}}{6}$.

139. $\frac{7 + 3\sqrt{5}}{2}$.

140. $\sqrt{2 + \sqrt{3}}$.

141. $\sqrt{10 + 7\sqrt{2}}$.

142. $\frac{a\,b^{\frac{1}{2}} - a^{\frac{1}{2}}\,b}{a - b}$.

143. $\frac{x^2\,y^{\frac{1}{2}} + x^{\frac{1}{2}}\,y}{x^3 - y}$.

144. $\frac{a + \sqrt{a^2 - x^2}}{x^2}$.

145. $\frac{\sqrt[3]{(a + \sqrt{a})\,(a^2 - a)^2}}{a^2 - a}$.

146. $\frac{\sqrt{x + 1} - \sqrt{x - 1}}{2}$.

147. $\frac{a + \sqrt{a^2 - x^2}}{x}$.

Art. 356. PAGE 218.

148. $x_1 = 25$
$x_2 = 36$.

149. $x_1 = 36$
$x_2 = 25$.

150. $x_1 = 9$
$x_2 = 6\frac{19}{25}$.
151. $x_1 = 256$
$x_2 = 625$.
152. $x_1 = 9$
$x_2 = (-17)^{\frac{2}{3}}$.
153. $x_1 = 343$
$x_2 = -\frac{125000}{343}$.
154. $x_1 = n^3$
$x_2 = -(n+1)^3$.
155. $x_1 = 196$
$x_2 = 49$.
156. $x_1 = 9$
$x_2 = 4$
$x_3 = -\frac{11}{2} + \frac{1}{2}\sqrt{-23}$
$x_4 = -\frac{11}{2} - \frac{1}{2}\sqrt{-23}$.
157. $x = 4$.
158. $x_1 = \frac{3}{2}$
$x_2 = \frac{2}{3}$.
159. $x_1 = 2\frac{1}{2}$
$x_2 = -2\frac{1}{2}$
$x_3 = \frac{2}{5}\sqrt{-1}$
$x_4 = -\frac{2}{5}\sqrt{-1}$.
160. $x_1 = 4$
$x_2 = 9$.
161. $x_1 = \frac{4}{9}$
$x_2 = -\sqrt[3]{\frac{1}{16}}$.
162. $x_1 = 256$
$x_2 = 256\sqrt[3]{6561}$.
163. $x = \left(\frac{a}{2} \pm \sqrt{b + \frac{a^2}{4}}\right)^{\frac{2n}{m}}$.

Art. 358. Page 220.

164. $x = 3$.
165. $x = 4$.
166. $x_1 = 171$
$x_2 = 144$.
167. $x_1 = 2$
$x_2 = -3$.
168. $x_1 = 3$
$x_2 = 1$.
169. $x = 4$.
170. $x = \frac{7}{12}$.
171. $x = 1$.
172. $x = 49$.
173. $x = 12$.
174. $x = 5$.
175. $x = 1\frac{1}{4}$.
176. $x_1 = 2\frac{1}{2}$
$x_2 = -2\frac{1}{2}$.
177. $x = 18a$.
178. $x = 2\frac{1}{8}$.
179. $x = \frac{1}{3}$.
180. $x = 30$.
181. $x = \frac{1}{2}$.
182. $x = 16$.
183. $x_1 = 7$
$x_2 = -\frac{559}{47}$.
184. $x_1 = 3$
$x_2 = 2$.
185.* $x_1 = a$
$x_2 = c$.
186. $z = 2a$.
187. $z = -2a$.
188. $x = \frac{9a}{16}$.
189. $x = -\frac{3}{4}$.
190. $y = 15\frac{61}{81}$.
191. $z = 7\frac{1}{5}$.
192. $x_1 = \sqrt{65}$
$x_2 = \sqrt{5}$.

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* The given equation may be reduced to the form $(x-a)(x-c)[(x-a)(x-c)-(x-b)(x-a+b-c)]=0$, then to the form $(x-a)(x-c)[ac-ab-bc+b^2]=0$, and finally, by dividing each member by the factor in square brackets, to $(x-a)(x-c)=0$.

SECTION XII.

Art. 366. Page 223.

1. $l = 298$, $S = 14950$. The series is 1, 4, 7 . . . 298.

2. $n = 13$, $S = 286$. The series is 40, 37, 34 . . . 4.

3. $a = -15$, $S = 120$. The series is -15, -12, -9 . . . 30.

4. $n = 8$, $d = -4$. The series is 20, 16, 12 . . . -8.

5. $l = -15$, $d = -5$. The series is 40, 35, 30 . . . -15.

6. $a = -12$, $d = 4$. The series is -12, -8, -4 . . . 20.

7. $a = 82$, $l = 118$. The series is 82, 86, 90 . . . 118.

8. $d = -2\frac{1}{2}$, $S = 157\frac{1}{2}$. The series is 30, $27\frac{1}{2}$, 25 . . . 15.

9. $l = 42$, $n = 12$. The series is 9, 12, 15 . . . 42.

10. $a = 1$, $n = 10$. The series is 1, 5, 9 . . . 37.

Art. 367. Page 223.

11. $106\frac{1}{2}$.

12. 58.

13. 5050.

14. 500500.

15. 2500.

16. 2550.

17. 370.

18. 6 or 12.

19. 6 or 9, 6 or -3.

20. 8 or 31.

21. $5996.25.

22. $16\frac{1}{12}t^2$; $6433\frac{1}{3}$ feet.

23. In 16 seconds.

24. $2n - 1$, and $2n$.

25. $\frac{n + n^2}{2}$.

26. n^2.

27. $n + n^2$.

28. $\frac{3n + n^2}{16}$.

29. $\frac{n - 1}{2}$.

30. 222.

31. $d = \frac{29}{40}$. $a = -10\frac{7}{20}$.
$l = 28\frac{4}{5}$. $S = 507\frac{3}{8}$.

32. $a = 5$. $d = 7$.

33. 5 prizes.

34. $a = 1$. $d = 3$.

35. $a = 25$, $d = -3$; or, $a = 4$, $d = 3$.

36. 10 days.

37. The average value of a number of quantities is found by dividing the sum of those quantities by the number of them.

38. The $(k-m)^{\text{th}}$ term is $a+(k-m-1)\,d$.
The $(k+m)^{\text{th}}$ term is $a+(k+m-1)\,d$.
The k^{th} term is $a+(k-1)\,d$.

39. $3, 3\frac{1}{2}, 4, 4\frac{1}{2} \ldots 16, 16\frac{1}{2}, 17$.

40. $10, 10\frac{10}{11}, 11\frac{9}{11}, 12\frac{8}{11} \ldots 18\frac{2}{11}, 19\frac{1}{11}, 20$.

41. $\dfrac{k-h}{m+1}$.

42. Let m be as much greater than a as it is less than b. Then $m-c=b-m$, whence $m=\dfrac{a+b}{2}$.

Art. 375. Page 228.

43. $l=-\frac{32}{625}$, $S=3\frac{348}{625}$. The series is $5, -2, \frac{4}{5}, -\frac{8}{25}, \frac{16}{125}, -\frac{32}{625}$.

44. $a=40$, $S=44.444$. The series is 40, 4, 0.4, 0.04, 0.004.

45. $a=5$, $l=\frac{5}{243}$. The series is $5, 1\frac{2}{3}, \frac{5}{9}, \frac{5}{27}, \frac{5}{81}, \frac{5}{243}$.

46. $r=\frac{2}{3}$, $S=6\frac{17}{108}$. The series is $\frac{9}{4}, \frac{6}{4}, 1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}$.

Art. 376. Page 230.

53. $r=\frac{2}{3}$, $S=9\frac{17}{72}$.

54. $l\,(280-l)^3=7\times 273^3=189\times 91^3$; whence $l=7$, or $l=189$, of which the latter is the answer to the problem. $r=3$. The series is 7, 21, 63, 189.

55. $r=\sqrt[10]{\frac{298977}{109260}}=1.1059$.* The yearly rate of *increase* was 0.1059.

56. $17\times(\frac{17}{20})^{28}=0.4047$ of a gallon.*

57. $\dfrac{(a-b)^n}{a^{n-1}}$.

58. $76\times(\frac{76}{96})^{24}=0.2791$ of a pound.*

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* This reduced value is best found by the use of logarithms. The arithmetical process would be too long.

59. $\dfrac{a^{n+1}}{(a+b)^n}$.

60. 27, 18, 12, 8.

61. 3, 5, 7, 9.

62. $a^n\,r^{\frac{1}{2}n(n-1)}$.

63. $\dfrac{1}{a}, \dfrac{1}{a\,r}, \dfrac{1}{a\,r^2} \ldots \dfrac{1}{a\,r^{n-1}}$ is a geometric progression in which the common ratio is $\dfrac{1}{r}$.

64. $a^2\,r, a^2\,r^3, a^2\,r^5 \ldots$ a geometrical progression in which the common ratio is r^2.

65. $a\,r \pm a, a\,r^2 \pm a\,r, a\,r^3 \pm r^2, a\,r^4 \pm r^3 \ldots$ a geometrical progression in which the common ratio is r.

66. The $(k-m)^{\text{th}}$ term is $a\,r^{k-m-1}$.
The $(k+m)^{\text{th}}$ term is $a\,r^{k+m-1}$.
The k^{th} term is $a\,r^{k-1}$.

67. $\frac{8}{27}, \frac{4}{9}, \frac{2}{3}, 1, 1\frac{1}{2}, 2\frac{1}{4}$.

68. $1, 2^{\frac{1}{12}}, 2^{\frac{2}{12}}, 2^{\frac{3}{12}} \ldots 2^{\frac{11}{12}}, 2$.

69. $\sqrt[m+1]{\dfrac{k}{h}}$.

70. The common ratio, $\sqrt{\dfrac{B}{A}}$. The series, $A, \sqrt{AB}, B$.

71. Compare the result of the last problem with Art. 196, and the truth is evident.

72. 24 and 6.

73. If a is the arithmetical mean, and b the geometrical mean, of two numbers, the numbers themselves are
$a+\sqrt{a^2-b^2}$ and $a-\sqrt{a^2-b^2}$.

Art. 378. Page 233.

74. $1\frac{1}{2}$.
75. $\frac{1}{4}$.
76. $\frac{1}{9}$.
77. $\frac{5}{33}$.
78. 2.
79. 54.
80. $\frac{1}{3}$.
81. $\frac{3}{4}$.
82. $\frac{1}{12}$.

83. At a point $\frac{1}{11}$ of a stadium beyond where the tortoise was at the outset.

Art. 380. Page 234.

84. $\frac{1}{9}$.
85. $\frac{1}{3}$.
86. $\frac{2}{3}$.
87. $\frac{6}{11}$.
88. $\frac{1}{300}$.
89. $\frac{1}{18}$.
90. $\frac{7}{12}$.
91. $\frac{5}{27}$.
92. $2\frac{59}{180}$.
93. $\frac{19}{101}$.
94. $2\frac{16}{37}$.
95. $\frac{7}{24}$.
96. $\frac{3}{7}$.

SECTION XIII.

Art. 405. Page 249.

31. 1.7324.
32. 5.9685.
33. 3.5911.
34. 4.9956.
35. 0.8388.
36. $\bar{1}.8921$.
37. 3.6990.
38. $\bar{4}.8976$.
39. $\bar{1}.4314$.
40. $\bar{5}.7782$.
41. 65000.
42. 33.
43. 6.7.
44. 430000.
45. 800.
46. 0.16.
47. 0.0047.
48. 0.00088.
49. 0.052.
50. 0.11.

Art. 407. Page 252.

51. 2.4579.
52. 1.7380.
53. 0.9996.
54. 6.5391.
55. $\bar{2}.6955$.
56. $\bar{4}.9943$.
57. 7720.
58. 9.89.
59. 0.0106.
60. 0.147.
61. 0.000417.
62. 0.00399.

Art. 411. Page 255.

63. 4.6877.
64. $\bar{2}.9927$.
65. 6.8632.
66. 6.9962.
67. $\bar{3}.1236$.
68. 0.0010.
69. 0.4753.
70. 1.7400.
71. 0.0004.
72. 3.0006.
73. 1.9929.
74. 0.0191.
75. 1.7488.
76. 1.9539.

Art. 414. Page 257.

77. 35540.
78. 0.004788.
79. 5.396.
80. 0.8892.
81. 709.4.
82. 0.10139.

83. 128350.
84. 0.06977.
85. 0.00010021.
86. 1015300.
87. 15604.
88. 3090.
89. 0.000010286.
90. 10.402.
91. 600.1.
92. 1.1015.
93. 8245000000.
94. 0.00000008054.

Art. 417. Page 259.

95. 852000000000.
96. 0.01458.
97. 48110.
98. 215.86.
99. 137220 feet.
100. 35810 cubic inches.

Art. 421. Page 261.

101. 1.975.
102. 6.77.
103. 31830.
104. 0.03866.
105. 4.009.
106. 1.1023.
107. 10.673.
108. 8.682.
109. 0.09852.
110. 0.037.
111. 0.02858.
112. 400.
113. 68.64.
114. 0.03113.
115. 5.334.

Art. 423. Page 263.

116. 14490000000000000000000.
117. 4055000000000000000000000.
118. 0.00001631.
119. 2.449.
120. 1.913.
121. 0.09486.
122. 0.9284.
123. 1.452.
124. 1.008.
125. 0.9245.
126. 9.200.
127. 0.02148.
128. 0.005776.
129. 0.15824.
130. 473.5
131. 0.009786.
132. 0.3016.
133. 1.0985.
134. 1.0251.
135. 2.69.

Art. 424. Page 264.

136. 0.1142.
137. 0.06789.
138. 0.000000000000000684.
139. 0.002011.
140. 0.5196.
141. 0.1368.
142. 1.252.
143. 0.9623.
144. 1.6801.
145. 0.7423.
146. 0.1459.
147. 0.8226.
148. 25.38.
149. 7.039.
150. 4.678.

151. 12.615.
152. 10.493.
153. 1.364.
154. 12.36.
155. 6.470.
156. 4.980.
157. 26.51.
158. 1.284.
159. 0.2243.
160. 83.87.
161. 14.556.
162. 13.152.
163. 2.515.

Art. 426. Page 266.

164. 2.861.
165. 1.337.
166. 0.6310.
167. 2.861.
168. 204.[*]
169. 28.07.
170. 36.60.
171. 11.90.

Art. 428. Page 269.

172. $735.59.
173. $1960.95.
174. $106.17.
175. $4.602.
176. $3069.57.
177. $13841.90.
178. $5\frac{1}{2}$ per cent.
179. 0.074.
180. 0.07177.
181. 0.02048.
182. 11.9 years.[†]
183. 28.07 years.[†]
184. 3.686 years.[†]
185. 9.12 years.[†]
186. 0.02516; that is, a little more than $2\frac{1}{2}$ per cent.
187. 64103000. 82183000.
188. [$n = 27.8976.$] In a little less than 28 years.
189. 0.1059; that is, about $10\frac{3}{5}$ per cent per annum.

Art. 431. Page 272.

190. $1206.83.
191. $5137.90.
192. $2100.20.
193. $2880.10.
194. $5366.40.

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[*] By seven-place table, 202.9356. In the rest of this section many of the answers are found by the use of a five-place or of a seven-place table. This is desirable when high powers of a number are to be used, since both the possible and the probable errors become very large. The first three figures of the answers may, however, be found by a four-place table in most cases.

[†] This is the value of n obtained from the formula, but is not a strictly accurate answer to the question. The exact answer could be found by computing the earnings of the given principal for the number of *whole years* found in the value of n, and then finding how large a fraction of a year would be required for the principal as it stood at the end of the last complete year to earn at *simple interest* the rest of the amount required to be earned. In Ex. 185 the computation would be made in a similar way for the number of entire half-years.

Art. 434. Page 275.

195. \$13489.

196. \$7805.50.

197. The pension; \$231.30 more valuable.

198. \$1631.20.

199. \$217,340.83.

200. [$n = 36.72.$] A fraction less than 37 years.

201. \$6419.41.

202. In a fraction less than 459 years. [$n = 458.6.$]

SECTION XIV.

Page 277.

1. \$17200.

2. A, \$1350. B, \$900.

3. A, 3200. B, 4800. C, 6000. D, 7000.

4. 24 days.

5. On the shorter arm, 400 pounds; on the longer, 350 pounds.

6. At $43\frac{7}{11}$ minutes past 2 o'clock.

7. 32 minutes past 9 o'clock.

8. 352 yards and 440 yards a minute respectively.

9. 52 yards.

10. 240.

11. $18\frac{2}{11}$ miles and $17\frac{1}{22}$ miles per hour respectively. The whole trip, $136\frac{4}{11}$ miles.

12. Estate \$81000. 9 children. Each, \$9000.

13. $\frac{a m n}{m + n}$.

14. $\frac{b m}{a}$.

15. $\frac{k l c}{100}$.

16. $\frac{b m}{a}$.

17. $\frac{a k' n}{a' k}$.

18. $\frac{a k}{b}$.

19. $\frac{a n - b m}{n - m}$.

20. $\frac{100(l - k)}{k}$.

21. 6.4 months.

22. $\dfrac{ak + bl + cm + dn}{a + b + c + d}$.

23. 510 inches.

24. 75 cents per bushel.

25. A, in $10\frac{2}{3}$ days.
B, in 32 days.

26. \$3000, at 4 %.

27. 45.94 kilograms.

28. $\dfrac{ad + bc}{2bd}$ and $\dfrac{ad - bc}{2bd}$.

29. 27 and 48.

30. As 56 to 143.

31. 5 : 1.

32. $2\frac{4}{15}$.

33. $\frac{9}{25}$.

34. $\frac{3}{4}$.

35. $\frac{5}{3}$.

36. $\dfrac{mpa}{mp + np + nq}$, $\dfrac{npa}{mp + np + nq}$, $\dfrac{nqa}{mp + np + nq}$.

37. $\dfrac{mpra}{mpr + npr + nqr + nqs}$, $\dfrac{npra}{mpr + npr + nqr + nqs}$, $\dfrac{nqra}{mpr + npr + nqr + nqs}$, $\dfrac{nqsa}{mpr + npr + nqr + nqs}$.

38. The pound, \$4.865. The franc, \$0.1925. The mark, \$0.2375.

39. 30, 48, and 50.

40. Of maple wood, 0.79. Of cork, 0.24. Of lead, 10.94.

41. $(x + 1)^2 - x^2 = (x + 1) + x$.

42. $(x - 1)^3 + x^3 + (x + 1)^3 = 3x^3 + 6x$, which is divisible by x.

43. $x - y = \dfrac{m}{n}$.

$m = n(x - y)$

$m(x + y) = n(x^2 - y^2)$.

44. 6, 9, and 18.

$\sqrt{ab}$, $\sqrt{bc}$, and $\sqrt{ca}$.

45. 15625. m^6.

46. Forwards, 4 miles an hour; backwards, 1 mile an hour.

47. 6 miles and 7 miles an hour respectively.

48. Fore wheel, 4 yards; hind wheel, 5 yards.

49. 100 or -80.

50. 6 inches.

51. 19.313 feet.

52. 5 sons.

53. The longer part, $-\frac{a}{2}+\frac{a}{2}\sqrt{5}$.

The shorter, $\frac{3a}{2}-\frac{a}{2}\sqrt{5}$. 6.180 and 3.820.

54. 5 miles an hour.

55. If r be the rate of rowing and s the rate of the stream, then either rate can be found from the other by means of the equation $br^2-2ar=bs^2$.

56. 35 and 10.

57. $\frac{a}{2}+\frac{a}{2}\sqrt{\frac{m-2}{m+2}}$ and $\frac{a}{2}-\frac{a}{2}\sqrt{\frac{m-2}{m+2}}$.

58. $\frac{\text{length}}{\text{breadth}}=2m^2-1\pm 2m\sqrt{m^2-1}$.

59. 24 and 30, or 8 and 10.

60. 6, 9, 10, and 15.

61. Base, 12. Perpendicular, 5. Hypothenuse, 13.

62. 36, 48, and 60.

Base, $\frac{1}{2}(\sqrt{h^2+4A}+\sqrt{h^2-4A})$.

Perpendicular, $\frac{1}{2}(\sqrt{h^2+4A}-\sqrt{h^2-4A})$.

63. Greater, $3\frac{3}{5}$. Less, $2\frac{2}{5}$.

64. Rectangle, $96(2\pm\sqrt{3})$ long, $32(2\pm\sqrt{3})$ wide.

Square, $32(3\pm 2\sqrt{3})$ each way.